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Pricing of options on futures with different price distributions

Shah Sarwar Rashid
Iowa State University

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Shah Sarwar Rashid

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1. INTRODUCTION

Black (1975) developed a formula for calculating the price of an option where the underlying asset is a futures contract. This was simply an extension of the option pricing formula of Black and Scholes (1973) where the underlying asset is stock. In both cases the assumptions used were as below.

- The continuous time return on the stock or on the futures for time period **t** is normally distributed.
- The mean of this normal distribution is proportional to **t**. The mean is assumed to be a constant multiplied by time which is μt.
- The variance of this normal distribution is proportional to **t**. The variance is assumed to be a constant multiplied by time which is $\sigma^2 t$.

Combining all the assumptions given above makes the price of the stock or the futures to have a lognormal distribution. There is significant evidence that the return on futures is not normally distributed and hence the price of the future is not lognormally distributed. This paper cites evidence in existing literature and shows empirical evidence of non-normality of the return on live hog futures.

Black used complicated mathematical concepts like diffusion process and Brownian motion to derive his formula for pricing option on futures. Also, as mentioned above as an assumption, the variability in the price for time period **t** was adjusted through the return structure instead of the variability in the price directly. This paper shows how, working only with the price of the futures with Lognormal probability distribution, Black's formula can be derived using simple mathematical and statistical tools.

Since there is sufficient evidence that the futures price may not be lognormally distributed, this paper shows the theoretical computation of pricing options on futures when the futures price has a Beta, Gamma or Uniform probability distribution.

This paper also discusses a simulation approach of pricing options on futures. Due to the open architecture of this simulation approach, option on futures with any probability distribution can be priced using this technique. If the current futures price and the variability in the futures price are known then, using those two known quantities and the assumed probability distribution of the futures price, the futures prices for any future time period can be generated by using any computer software e.g. EXCEL. It then computes the price of the option by only using those prices of the futures. This Simulation Approach requires the understanding of the mechanism of option pricing but doesn't require the understanding of difficult mathematical or statistical concepts. This paper shows the pricing of options on futures with Lognormal, Beta, Gamma or Uniform probability distribution by using this Simulation Approach. It also shows that, when the distribution of the futures price is assumed to be lognormal, the Simulation Approach gives the same result as gives Black's option pricing formula of options on futures.

1.1 Options

An option is a contract that gives the buyer the right to buy or sell an asset at a specified price within a specified period of time. Options are of two types, call and puts option. A call option gives the buyer the right to buy a certain asset by a certain date for a certain price. A put option gives the buyer the right to sell a certain asset by a certain date for a certain price. The price in the contract is called the strike price or exercise price. The date in the contract is called the expiration date or the exercise date or the date of maturity. The underlying asset of an options contract can be anything but usually they are bonds, commodities, foreign currencies, interest rates, stocks, stock indices etc. The exchange-traded options for agricultural commodities are options on futures of agricultural commodities. The value of the option mainly, not totally, depends on the value of the underlying asset.

There are three types of Options: American Options, European Options and Asian Options. An American Option can be exercised any time within the expiration date whereas a European Option can be exercised only on the expiration date. An Asian Option is one where underlying asset is based on the average price of some assets.

1.2 Pricing of Options on Stocks

An option on a particular stock gives the buyer the right to buy or sell, depending on whether it's a call or a put option, that particular stock at a specified price within a specified period of time. A stock option pricing model must make some assumptions regarding the movement of stock prices over time.

Bachelier (1900) first assumed that the stock prices were random variables and price changes were independently and identically distributed. He introduced the concept of Arithmetic Brownian Motion. Osborne (1959) was the first to rediscover normal distribution and Brownian Motion as a model for stock returns after the very well known work of Bachelier (1900). He worked with the stock price data from New York Stock Exchange (NYSE) and American Stock Exchange (ASE). He concluded the lognormal behavior of stock returns. Samuelson (1965) introduced the geometric Brownian Motion that gave the price process an exponential form.

Black and Scholes (1973) finally came up with an explicit formula for arbitrage-free pricing of a call option. The Black-Scholes model used the assumption that stock prices follow a random walk in continuous time with a variance rate proportional to the square of the stock price. This implies that the distribution of possible stock prices at the end of any finite interval is lognormal. In other words, the stock price follows a Geometric Brownian Motion through time which produces a lognormal distribution for stock price between any two points in time. Equivalently, it assumes that the continuously compounded rate of return on the stock in any given time interval is normally distributed.

A great deal of effort has been put into, and a great amount of empirical research has been done to refine Black-Scholes model and to identify differences between option prices calculated from the model and option prices observed in the market. Hull and White (1987) have shown that, the option price is lower than the Black-Scholes price when the option is close to being at the money and higher when it is deep in or deep out of the money. The exercise prices for which Black-Scholes model overprices the options are within about ten percent of the security price which are the range of exercise prices over which most option trading takes place. They also have claimed that, in general, the Black-Scholes model overprices options and this problem gets exaggerated as the time to maturity increases.

Tests of option pricing frequently show that in-the-money and out-of-the-money options appear to be mispriced relative to at-the-money options when using Black-Scholes model to price options. The volatility for which the Black-Scholes model correctly prices at-the-money options causes it to misprice in-the-money and out-of-the-money options. These pricing errors can be explained by differences between the lognormal distribution assumed by Black-Scholes model and the true distribution. There may be four ways in which the true asset price distribution can be different from the lognormal distribution but still give the same mean and standard deviation:

- The distribution with both tails thinner than that in lognormal distribution. For such distribution, Black-Scholes model overprices both out-of-the money and in-the-money calls and puts.
- The distribution with left tail fatter and right tail thinner than that in lognormal distribution. For such distribution, Black-Scholes model overprices out-of-the money calls and in-the-money puts. It underprices out-of-the money puts and in-the-money calls.
- The distribution with left tail thinner and right tail fatter than that in lognormal distribution. For such distribution, Black-Scholes model overprices out-of-the money puts and in-the-money calls. It underprices out-of-the money calls and in-the-money puts.
- The distribution with both tails fatter than that in lognormal distribution. For such distribution, Black-Scholes model underprices both out-of-the money and in-the-money calls and puts.

Mandelbrot (1963) and Fama (1965) have shown that the unconditional distribution of first differences of the logarithm of stock prices generally has fatter tails than a normal distribution has and that the error variances tend to cluster together. It otherwise means that stock returns have some distribution other than normal and consequently stock prices have some distribution other than lognormal.

Statistical and empirical analysis of data from financial markets, such as stock prices, interest rates or foreign exchange rates, shows that generalized hyperbolic (GH) distributions allow a more realistic description of returns than the classical normal distribution. Eberlein and Keller (1995) and Eberlein, Keller and Prause (1998) have shown that the return of stocks follow a heavier-tailed distribution than the normal distribution and the return can be modeled better by hyperbolic distribution than normal distribution. The hyperbolic distribution was introduced by Barndorff-Nielsen (1977) for modeling the grain-size distribution of windblown sand. The log density of the normal distribution is a parabola and the name "hyperbolic distribution" derives from the fact that the log density of such a distribution is a hyperbola.

The normal probability density, standard normal probability density, hyperbolic probability density and the option pricing formula with standard normal or hyperbolic return structure is shown below.

Normal Density: $f_{(\mu,\sigma)}(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ where, $-\infty \le \mu \le \infty$ and $\sigma \ge 0$

Standard Normal Density: $f(x) = \dfrac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$ special case where, $\mu = 0$ and $\sigma = 1$

Hyperbolic Density: $f_{(\alpha,\beta,\delta,\mu)}(x) = \dfrac{\sqrt{\alpha^2 - \beta^2}}{2\alpha\delta K_1(\delta\sqrt{\alpha^2 - \beta^2})} e^{-\alpha\sqrt{\delta^2 + (x-\mu)^2} + \beta(x-\mu)}$

where, K_1 denotes the modified Bessel function of the third kind.

$-\infty \le \mu \le \infty$, $\delta > 0$ and $0 \le |\beta| < \alpha$

$C_{normal} = S_0 \int_{\gamma}^{\infty} f(x)dx - Ke^{-rT} \int_{\gamma+\sigma}^{\infty} f(x)dx$ where, $\gamma = \dfrac{1}{\sigma} \ln(K/S0) - \dfrac{1}{2}\sigma$

and f(x) is the above-mentioned standard normal density.

$C_{hyperbolic} = S_0 \int_{\gamma}^{\infty} f_T(x;\theta+1)dx - Ke^{-rT} \int_{\gamma}^{\infty} f_T(x;\theta)dx$ where, $\gamma = \ln(K/S_0)$ and $f_t(.\,;\theta)$ is the above-mentioned hyperbolic density.

The hyperbolic distribution has fatter tails than has lognormal distribution. Eberlein and Keller (1995) worked with European Call Option for Deutsche Bank and showed that Black-Scholes model underpriced both out-of-the money and in-the-money calls but overpriced at-the money calls compared to the hyperbolic model.

1.3 Pricing of Options on Futures

An option on a particular futures gives the buyer the right to buy or sell, depending on whether it's a call or put option, that particular futures contract at a specified price within a specified period of time. Both the futures contract and options on futures are used more for agricultural commodities than for any other assets. An option on a futures contract where the underlying asset is an agricultural commodity is called to be commodity options. Commodity options are agreements that give buyers the right to buy or sell predetermined quantities of specified futures commodities at a fixed price within a predetermined period. A speculator is a market participant who has positions only in the options markets whereas a hedger is a market participant who has positions in the cash market, forward or futures market and in the options market.

Pricing option on futures is not that very different from pricing option on stocks. Some assumptions regarding the distribution of the futures price have to be made. Keynes (1930) first came up with the form of return that speculators earn from the futures market. He argued that speculators act as insurance providers who underwrite the risks of price fluctuation but the hedgers definitely pay for such prices. Thus he argued that futures prices are downward biased estimates of expected prices which mean that futures prices are less than the expected value of the spot price. Hardy (1940) argued that for many speculators the futures market is like a casino and hence they are willing to pay for the gamble. Thus futures prices are upward biased estimates of expected spot prices. Lester (1960) had a view similar to that of Hardy but opposite to that of Keynes. He argued that if speculators are sellers of insurance then they should make money on average but if they are buyers of gambles then they should loose money on average. He argued that there are no reasons for any consistent difference between the futures price and the spot price expected upon the expiration of the futures contract. As the speculators are buyers of gamble, there will be influx of new and fresh gamblers if there is a price differential between futures and expected spot prices. Thus the competition and free entry will bring the speculative profits to zero. He showed that speculators on average loose money.

Mandelbrot (1963) worked with cotton prices and showed that commodity and commodity futures prices can better be characterized by a Stable Paretian Distribution instead of a Gaussian or Normal distribution. He also mentioned that his theory could be applied to wheat and other edible grains. The logarithm of the characteristic function for the stable Paretian family of distribution is as below:

$$\text{Log } f(t) = \log \int_{-\infty}^{\infty} \exp(iut)dP(\tilde{u} < u) = i\delta t - \gamma|t|^{\alpha} [1 + i\beta(t/|t|) \tan(\alpha\pi/2)$$

The logarithm of the characteristic function for the normal distribution is

$$\log f(t) = i\mu t - (\sigma^2/2)*t^2$$

In the above-mentioned model the characteristic exponent α determines the height or total probability contained in the extreme tails of the distribution. It can take any value in the interval $0 < \alpha < 2$. The stable Paretian distribution becomes the Normal distribution when α has a value of 2. When α is in the interval $0 < \alpha < 2$, the extreme tails of the Paretian distributions are higher than those of the normal distribution and the area of the tail increases as α moves towards 0. The Paretian distribution only has finite mean but the variance and other moments are infinite for $\alpha \neq 2$.

Dusak (1973) worked with wheat, corn and soybean and concluded that the distributions of returns on futures contracts conform better to the stable non-Gaussian family than to the normal distribution. It seems that she agreed to the findings of Mandelbrot (1963). Her sample gave her both mean returns and systemic risk to be zero and hence she didn't find any contradiction between the capital market or Keynes approach and Hardy gambling Casino theory.

Black (1976) finally came up with an explicit formula for calculating the value of an option on futures contract. This was basically a slight modification of the Black-Scholes model. He showed that futures contracts are nothing but a series of forward contracts rewritten every day. He assumed that expected changes in the futures prices are zero. Thus the expected futures price at any time t' in the future, where t' is any time between current time t and the transaction time t^*, will be equal to the current futures price. He also assumed that the fractional change in the futures price over any interval is log-normally distributed. Combining all the facts, he assumed that the distribution of the futures price at any time in future has a lognormal distribution with mean equal to the futures price of today.

The literature on the distribution of the futures price suggests that the assumption of Brownian Motion or the Lognormal distribution is more questionable for the futures contract on commodity than any other futures contracts or stocks or assets. Stevenson and Bear (1970) claimed that more rigorous testing of random walk or Brownian motion hypothesis has been done in stock markets than in commodity markets. Thus the existing literature suggests that it is worthwhile to test the random walk assumption of futures prices of agricultural commodities. The next section discusses the empirical evidence of non-normality of return on the live hog futures.

2. EMPIRICAL EVIDENCE OF NON-NORMALITY

The live hog futures data were worked with in order to examine the behavior or the probability distribution of return on the futures. The data set had daily quotes of live hog futures contracts for the years 1970 to 1997 for February, April, June and July contracts and for the years 1970 to 1996 for August, October and December contracts.

2.1 Initial Data and Analysis

Daily price changes, from the previous trading day, of the futures were calculated from the daily price quotes. Thus the data set had 8069 observations of daily price changes for the February contracts, 8327 observations of daily price changes for the April contracts, 8342 observations of daily price changes for the June contracts, 8012 observations of daily price changes for the July contracts, 7438 observations of daily price changes for the August contracts, 7820 observations of daily price changes for the October contracts and 8219 observations of daily price changes for the December contracts. Altogether the data set had 56227 observations of daily price changes for all the contracts and it is described below as AllContracts.

For all the observations on the February contracts, summary statistics have been presented in Table 1, the histogram with normal curve of this data has been presented in Figure 1 and the normal probability plot of this data has been presented in Figure 2. For all the observations on the April contracts, summary statistics have been presented in Table 2, the histogram with normal curve of this data has been presented in Figure 3 and the normal probability plot of this data has been presented in Figure 4. For all the observations on the

Table 1. Statistics of the daily price changes on February contracts

Variable	N	Mean	Median	TrMean	StDev
FebPriceChange	8069	0.01710	0.00000	0.02000	0.76130

Variable	SE Mean	Minimum	Maximum	Q1	Q3
FebPriceChange	0.00848	-2.22750	2.29500	-0.33750	0.40500

Observations = 8069

Sample Mean	=	0.01710050936	Variance	=	0.579581
Standard Error	=	0.76130222107	SE of Sample Mean	=	0.008475
t-Statistic	=	2.01772	Signif Level (Mean=0)	=	0.04365303
Skewness	=	-0.06469	Signif Level (Sk=0)	=	0.01770705
Kurtosis	=	1.09117	Signif Level (Ku=0)	=	0.00000000

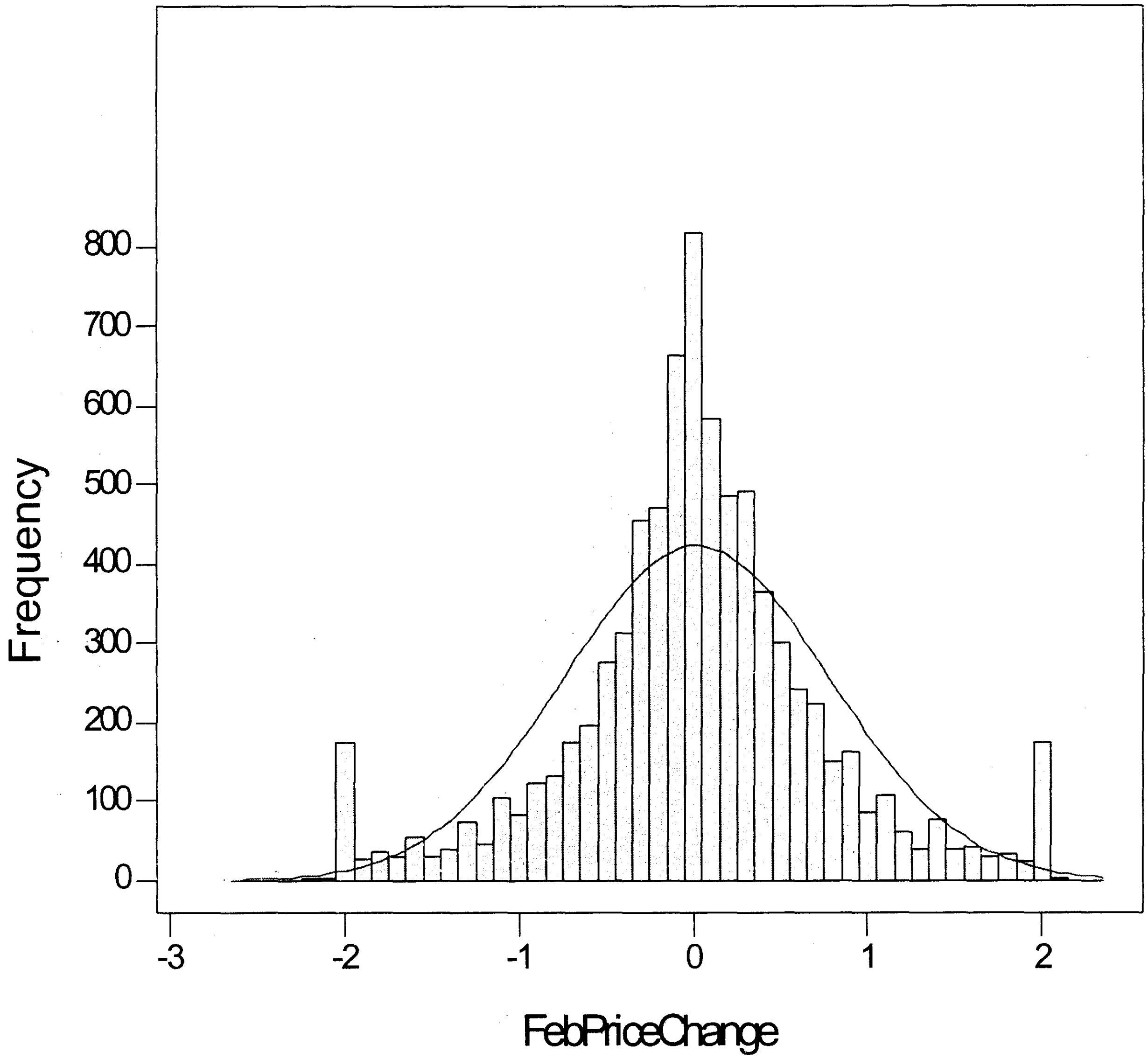

Figure 1. Histogram with normal curve for daily price changes on February contracts

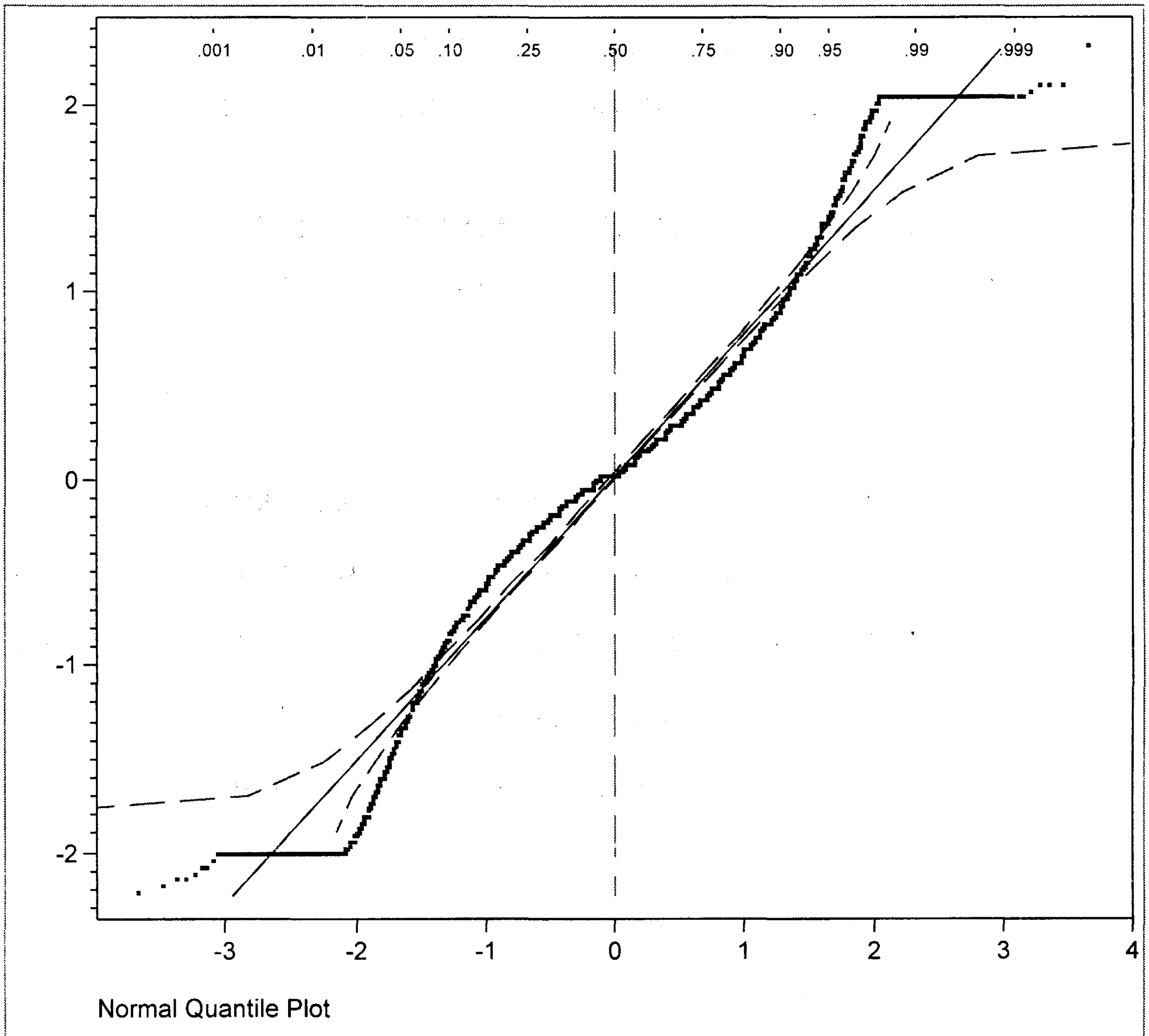

Figure 2. Normal probability plot for daily price changes on February contracts

Table 2. Statistics of the daily price changes on April contracts

Variable	N	Mean	Median	TrMean	StDev
AprPriceChange	8327	0.00988	0.00000	0.01286	0.76118

Variable	SEMean	Minimum	Maximum	Q1	Q3
AprPriceChange	0.00834	-2.70000	3.30750	-0.33750	0.37800

Observations = 8327

Sample Mean	=	0.00988309115	Variance	=	0.579399
Standard Error	=	0.76118242591	SE of Sample Mean	=	0.008342
t-Statistic	=	1.18481	Signif Level (Mean=0)	=	0.23612681
Skewness	=	-0.05350	Signif Level (Sk=0)	=	0.04628199
Kurtosis	=	1.08464	Signif Level (Ku=0)	=	0.00000000

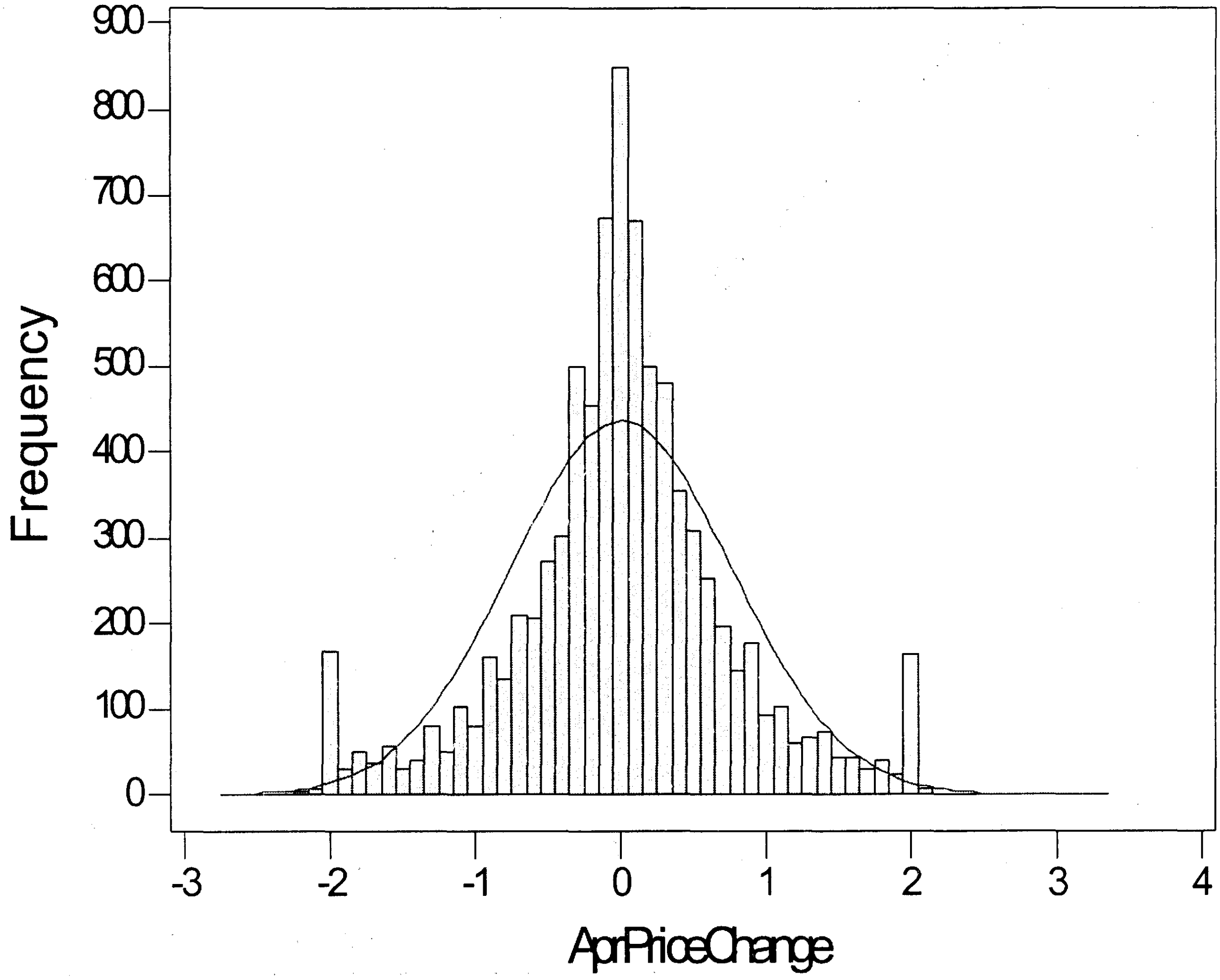

Figure 3. Histogram with normal curve for daily price changes on April contracts

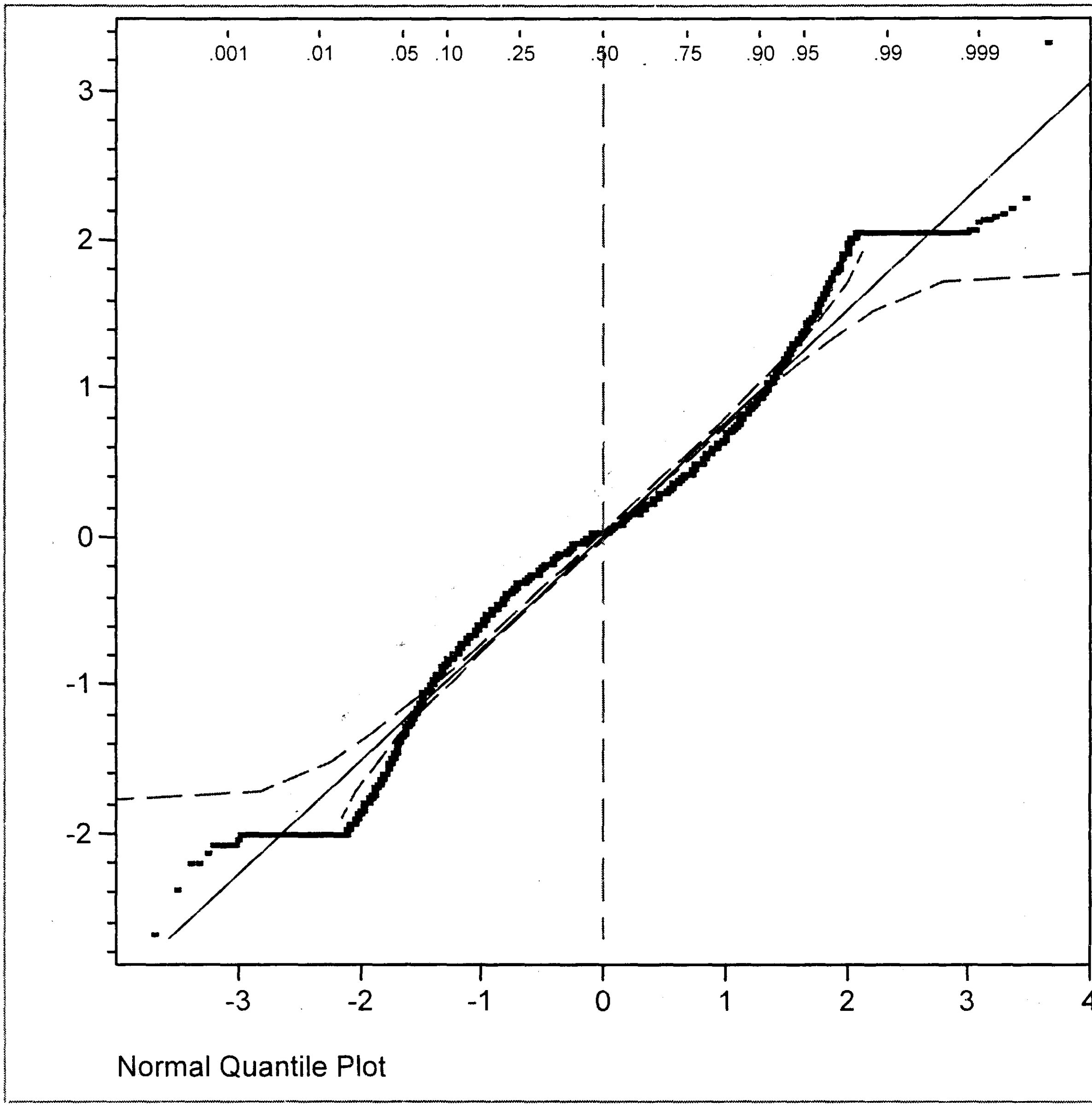

Figure 4. Normal probability plot for daily price changes on April contracts

June contracts, summary statistics have been presented in Table 3, the histogram with normal curve of this data has been presented in Figure 5 and the normal probability plot of this data has been presented in Figure 6. For all the observations on the July contracts, summary statistics have been presented in Table 4, the histogram with normal curve of this data has been presented in Figure 7 and the normal probability plot of this data has been presented in Figure 8. For all the observations on the August contracts, summary statistics have been presented in Table 5, the histogram with normal curve of this data has been presented in Figure 9 and the normal probability plot of this data has been presented in Figure 10. For all the observations on the October contracts, summary statistics have been presented in Table 6, the histogram with normal curve of this data has been presented in Figure 11 and the normal probability plot of this data has been presented in Figure 12. For all the observations on the December contracts, summary statistics have been presented in Table 7, the histogram with normal curve of this data has been presented in Figure 13 and the normal probability plot of this data has been presented in Figure 14. For all the observations on AllContracts, summary statistics have been presented in Table 8, the histogram with normal curve of this data has been presented in Figure 15 and the normal probability plot of this data has been presented in Figure 16. Under usual assumptions, the distribution of daily price changes should be normal with mean zero and constant variance.

If a distribution is normal, it should have the following properties.

- Histogram should be symmetric and bell-shaped or mound-shaped.
- Skewness, which measures the lack of symmetry in the distribution, should be zero.
- Kurtosis, which measures the peakedness or flatness of the distribution, should be three.
- Normal probability plot should look like a straight line.

Table 3. Statistics of the daily price changes on June contracts

Variable	N	Mean	Median	TrMean	StDev
JunPriceChange	8342	0.02777	0.00000	0.02836	0.77277

Variable	SE Mean	Minimum	Maximum	Q1	Q3
JunPriceChange	0.00846	-2.13301	2.70000	-0.33750	0.40500

Observations = 8342

Sample Mean	=	0.02776624431	Variance	=	0.597178
Standard Error	=	0.77277318692	SE of Sample Mean	=	0.008461
t-Statistic	=	3.28171	Signif Level (Mean=0)	=	0.00103604
Skewness	=	-0.01276	Signif Level (Sk=0)	=	0.63436097
Kurtosis	=	0.96443	Signif Level (Ku=0)	=	0.00000000

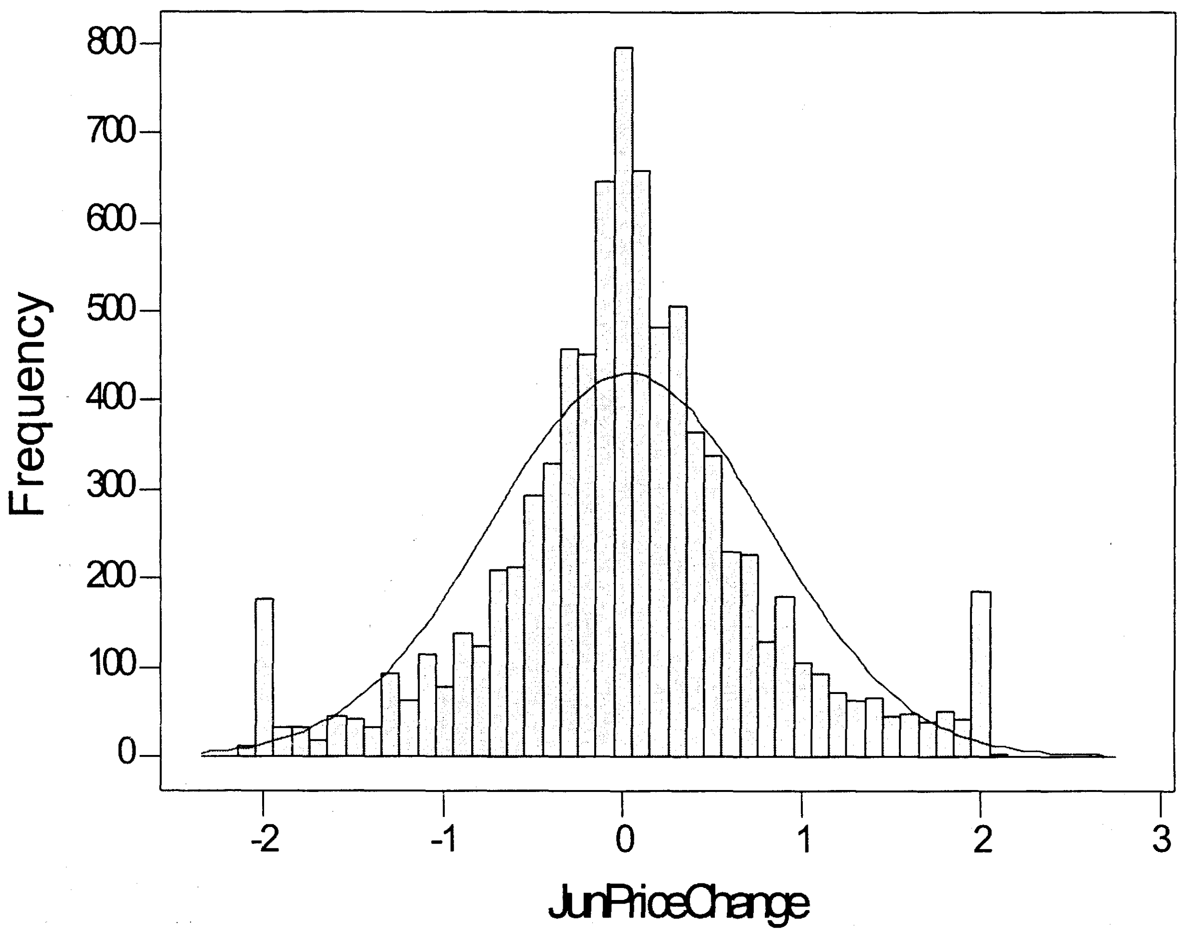

Figure 5. Histogram with normal curve for daily price changes on June contracts

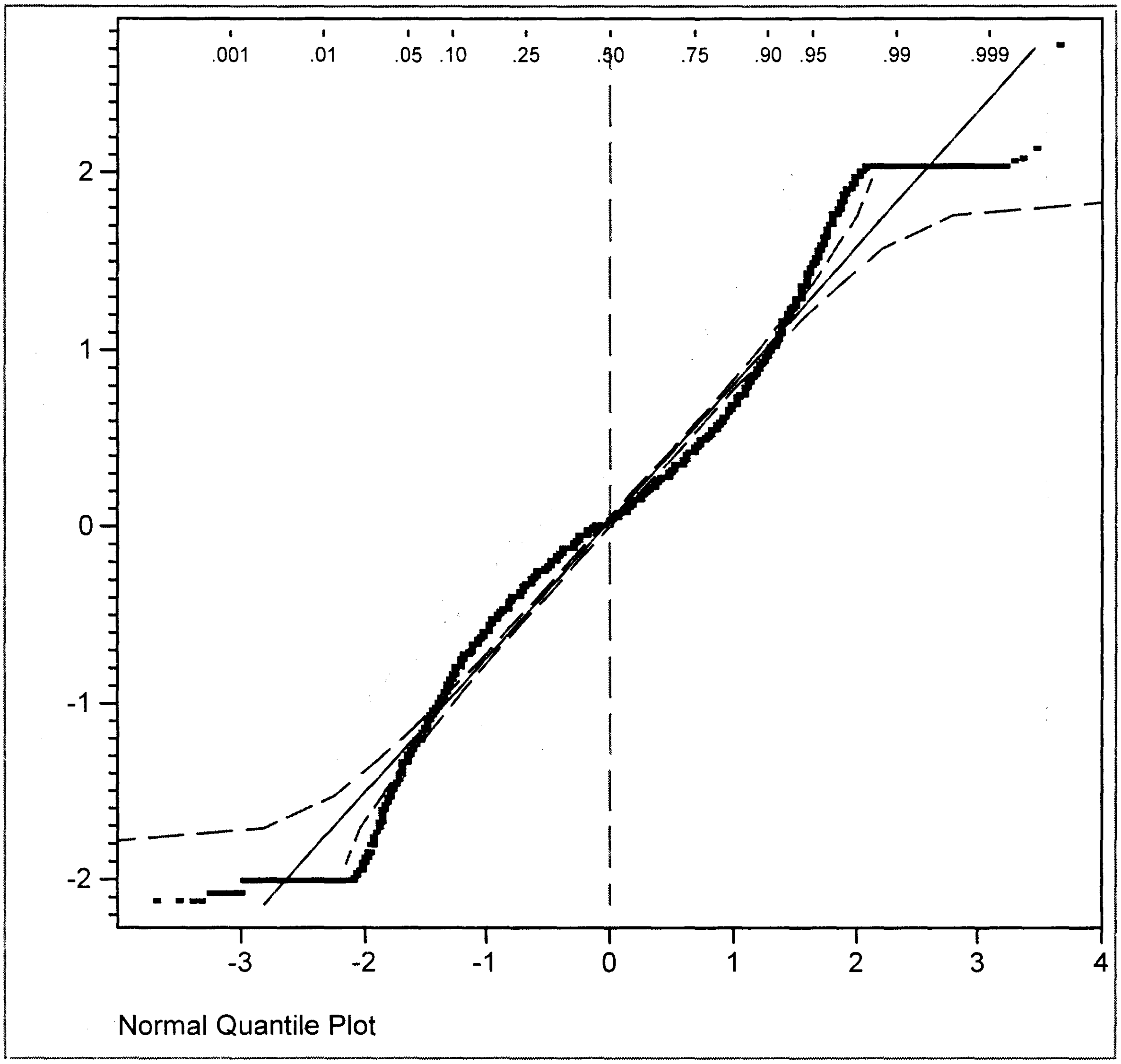

Figure 6. Normal probability plot for daily price changes on June contracts

Table 4. Statistics of the daily price changes on July contracts

Variable	N	Mean	Median	TrMean	StDev
JulPriceChange	8012	0.02775	0.00000	0.02935	0.78217

Variable	SE Mean	Minimum	Maximum	Q1	Q3
JulPriceChange	0.00874	-2.70000	2.49751	-0.33750	0.43200

Observations = 8012

Sample Mean	=	0.02775193460	Variance	=	0.611788
Standard Error	=	0.78216896924	SE of Sample Mean	=	0.008738
t-Statistic	=	3.17587	Signif Level (Mean=0)	=	0.00149953
Skewness	=	-0.03938	Signif Level (Sk=0)	=	0.15023259
Kurtosis	=	0.89194	Signif Level (Ku=0)	=	0.00000000

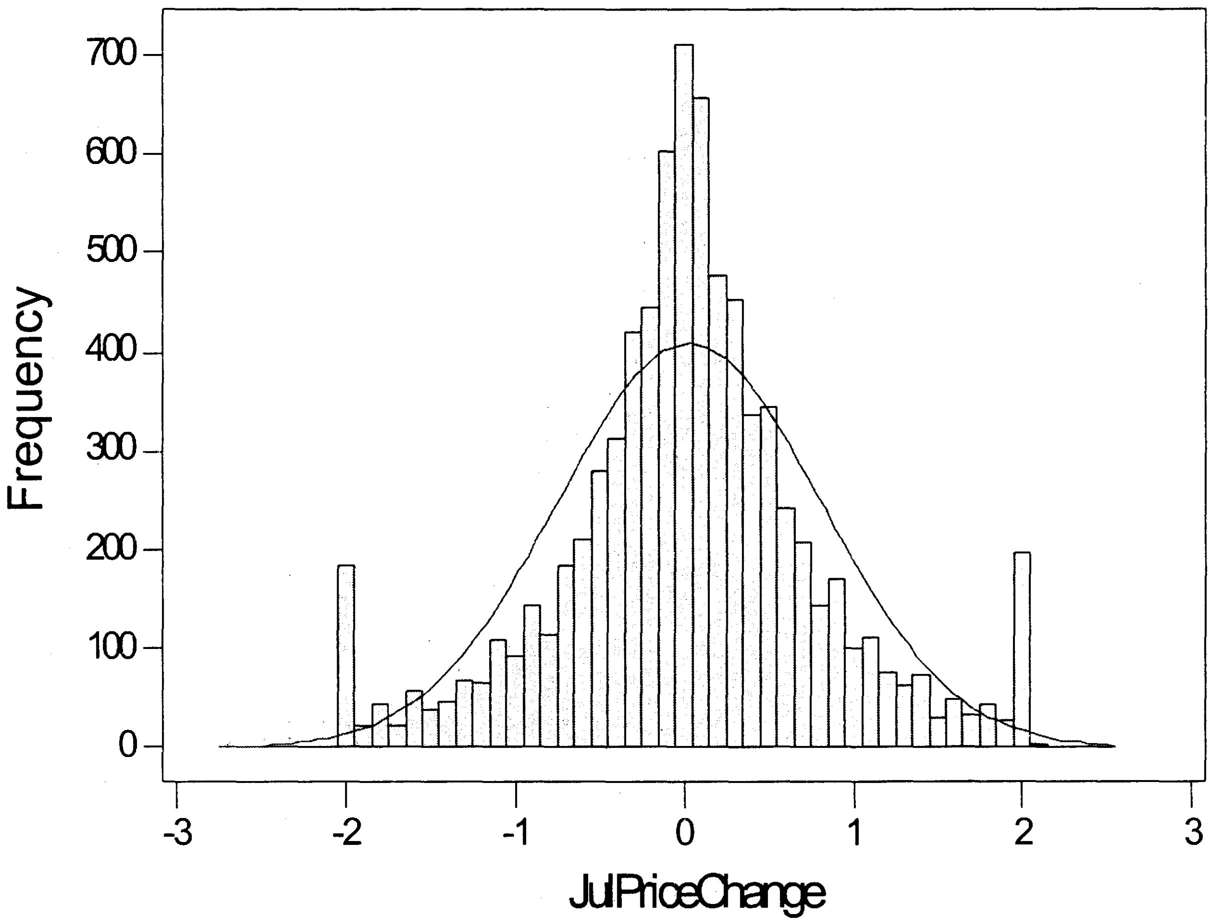

Figure 7. Histogram with normal curve for daily price changes on July contracts

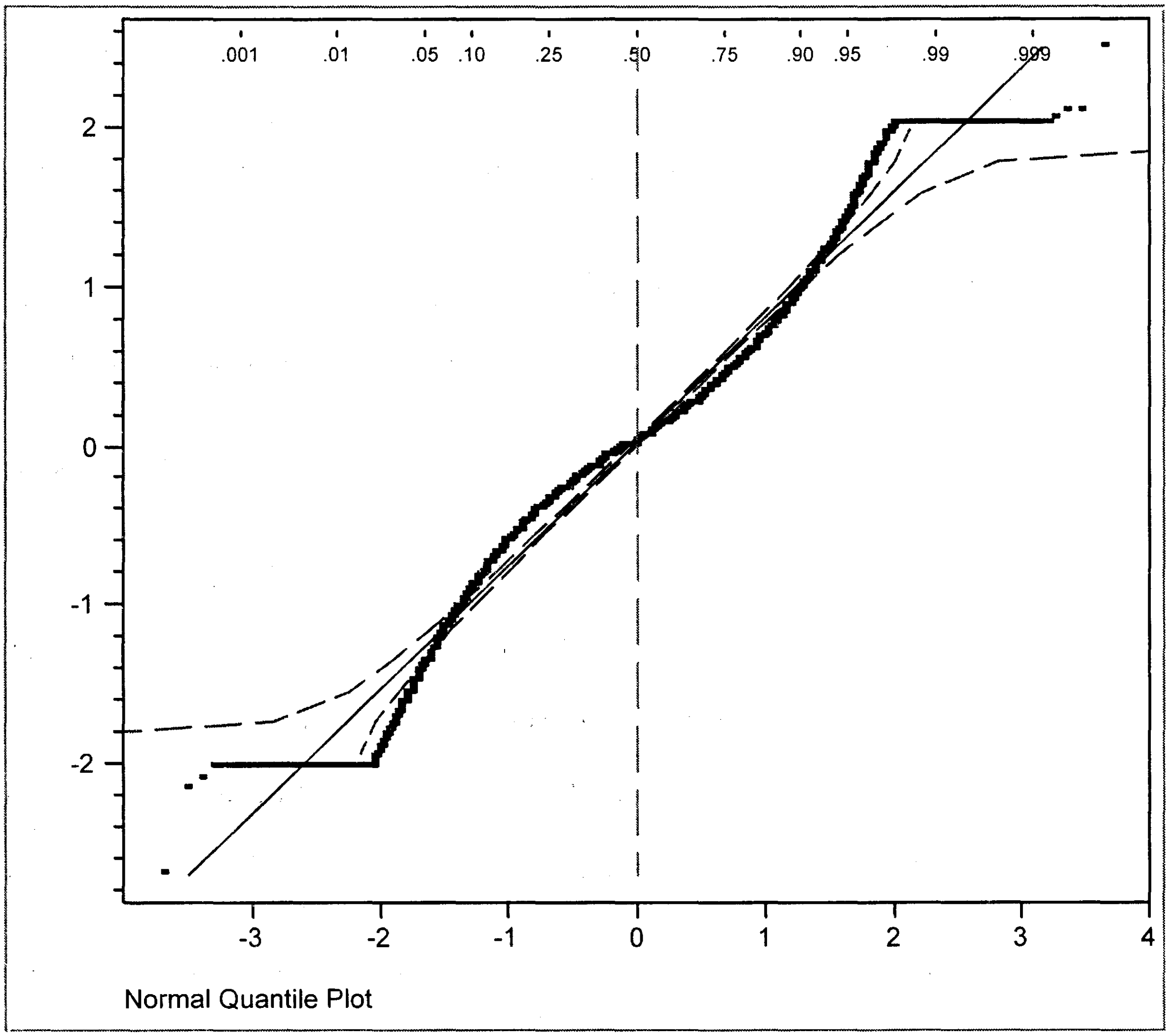

Figure 8. Normal probability plot for daily price changes on July contracts

Table 5. Statistics of the daily price changes on August contracts

Variable	N	Mean	Median	TrMean	StDev
AugPriceChange	7438	0.02609	0.02700	0.02968	0.79964

Variable	SE Mean	Minimum	Maximum	Q1	Q3
AugPriceChange	0.00927	-3.78000	4.18500	-0.33750	0.40500

Observations = 7438

Sample Mean	=	0.02608524066	Variance	=	0.639417
Standard Error	=	0.79963538289	SE of Sample Mean	=	0.009272
t-Statistic	=	2.81340	Signif Level (Mean=0)	=	0.00491498
Skewness	=	-0.08126	Signif Level (Sk=0)	=	0.00422754
Kurtosis	=	0.98374	Signif Level (Ku=0)	=	0.00000000

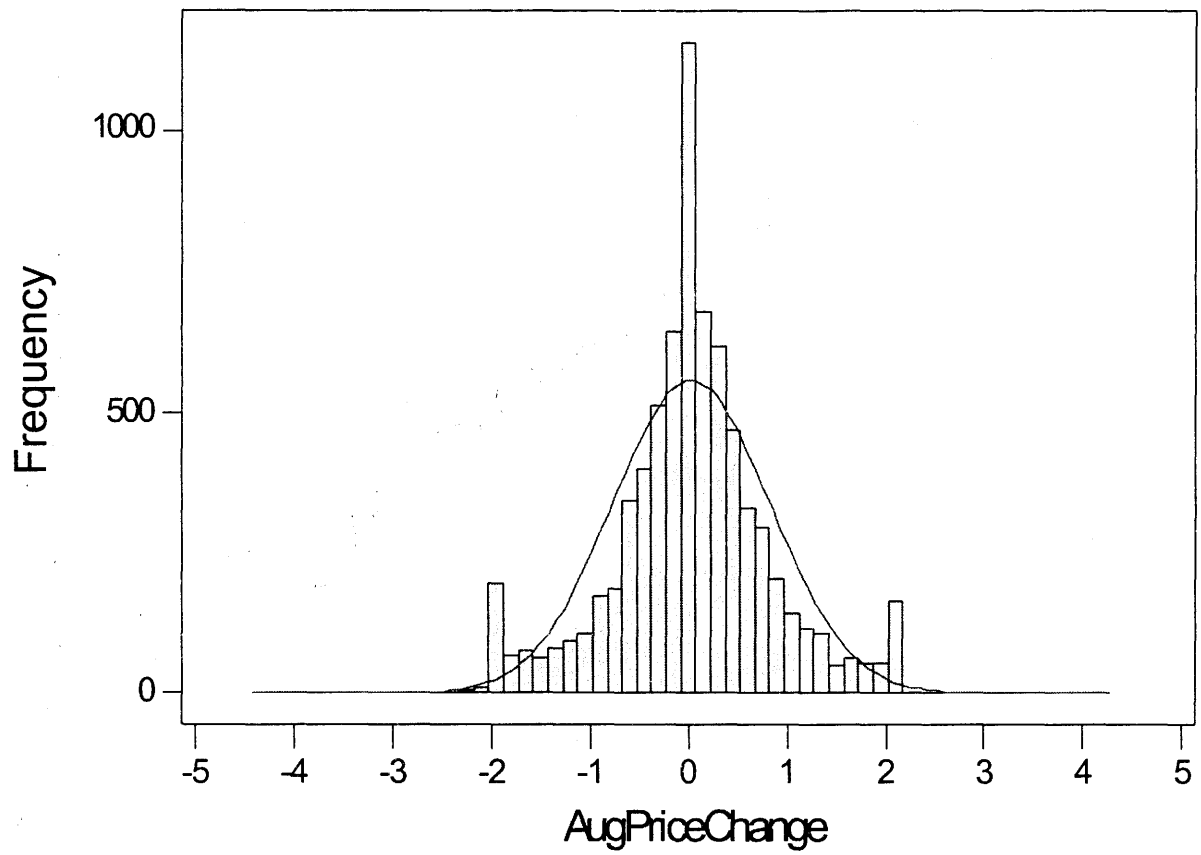

Figure 9. Histogram with normal curve for daily price changes on August contracts

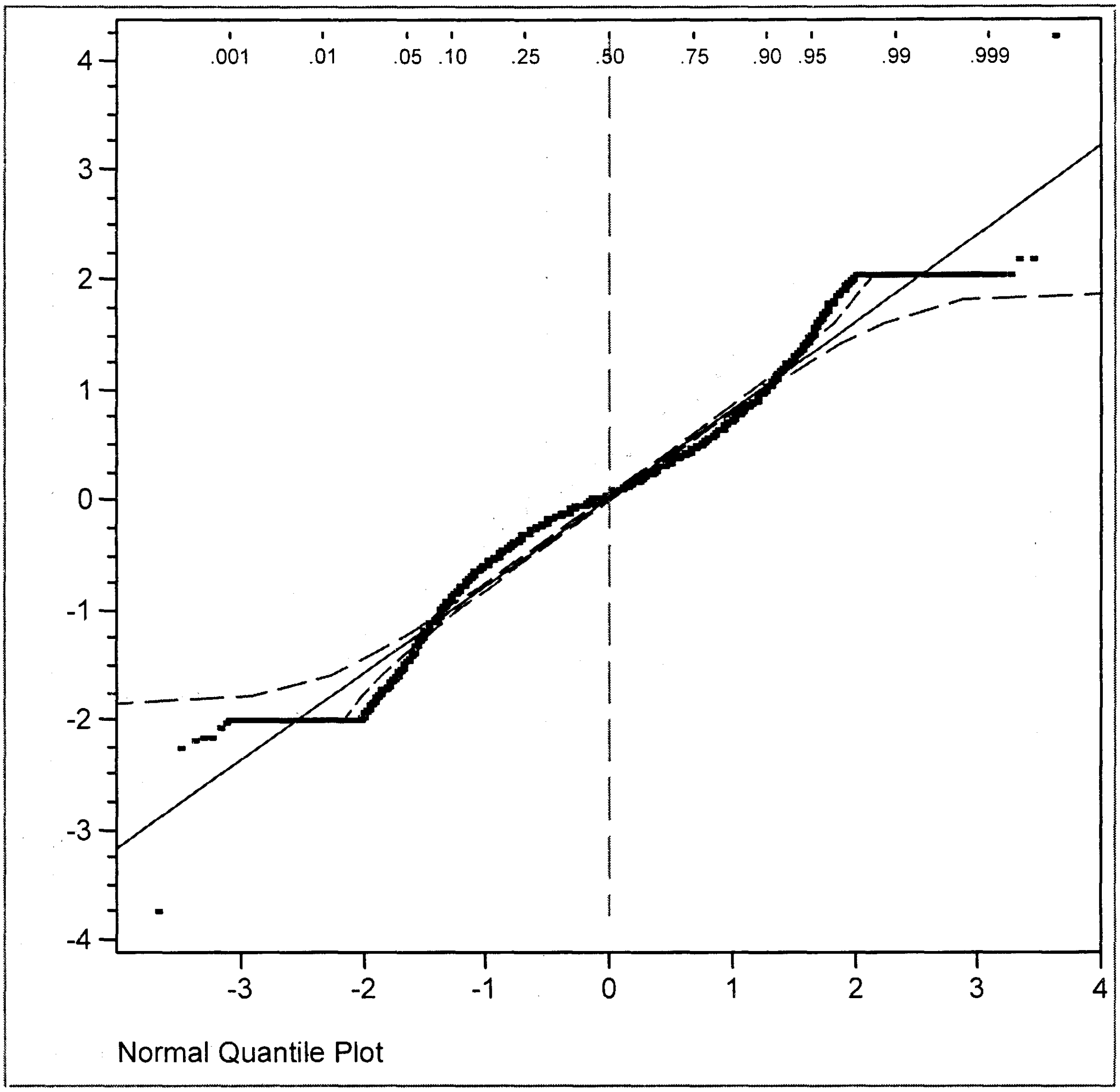

Figure 10. Normal probability plot for daily price changes on August contracts

Table 6. Statistics of the daily price changes on October contracts

Variable	N	Mean	Median	TrMean	StDev
OctPriceChange	7820	0.01705	0.00000	0.01836	0.75568

Variable	SE Mean	Minimum	Maximum	Q1	Q3
OctPriceChange	0.00855	-2.20050	2.16000	-0.33750	0.36450

Observations = 7820

Sample Mean	=	0.01705108696	Variance	=	0.571054
Standard Error	=	0.75568106938	SE of Sample Mean	=	0.008545
t-Statistic	=	1.99534	Signif Level (Mean=0)	=	0.04604045
Skewness	=	-0.02912	Signif Level (Sk=0)	=	0.29316402
Kurtosis	=	1.15950	Signif Level (Ku=0)	=	0.00000000

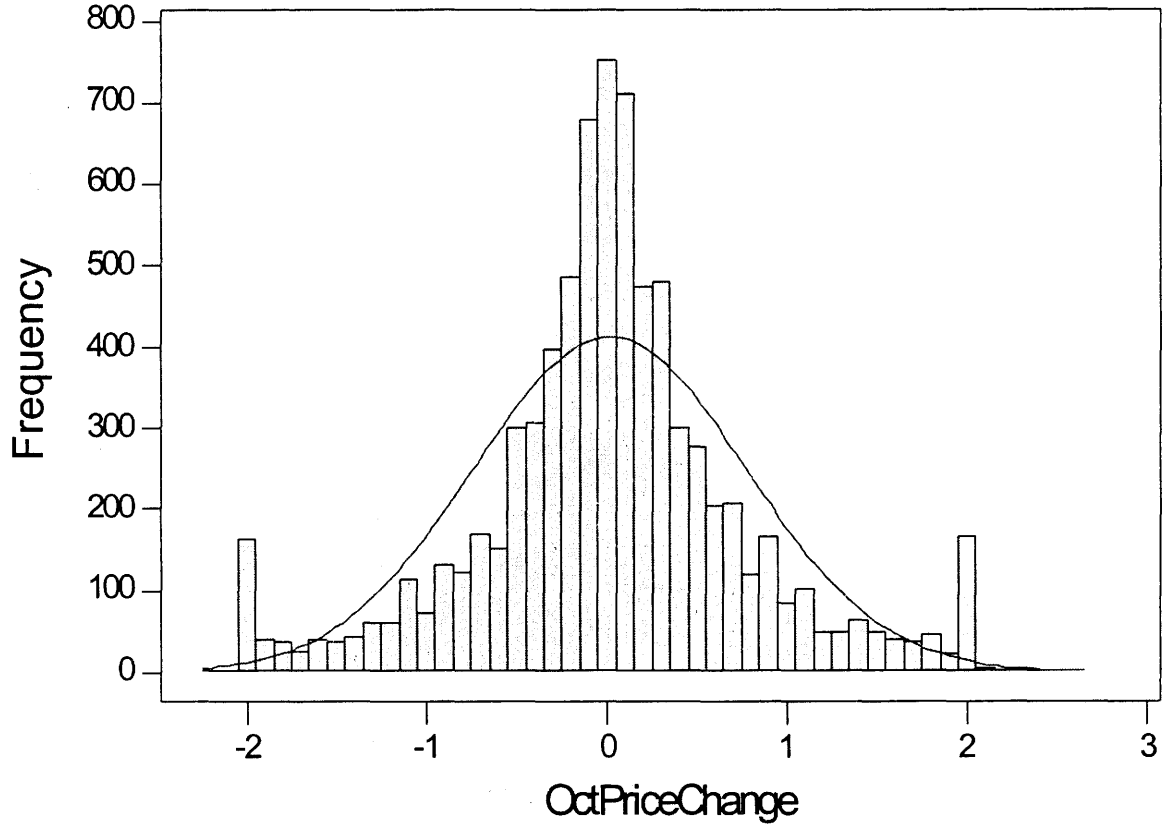

Figure 11. Histogram with normal curve for daily price changes on October contracts

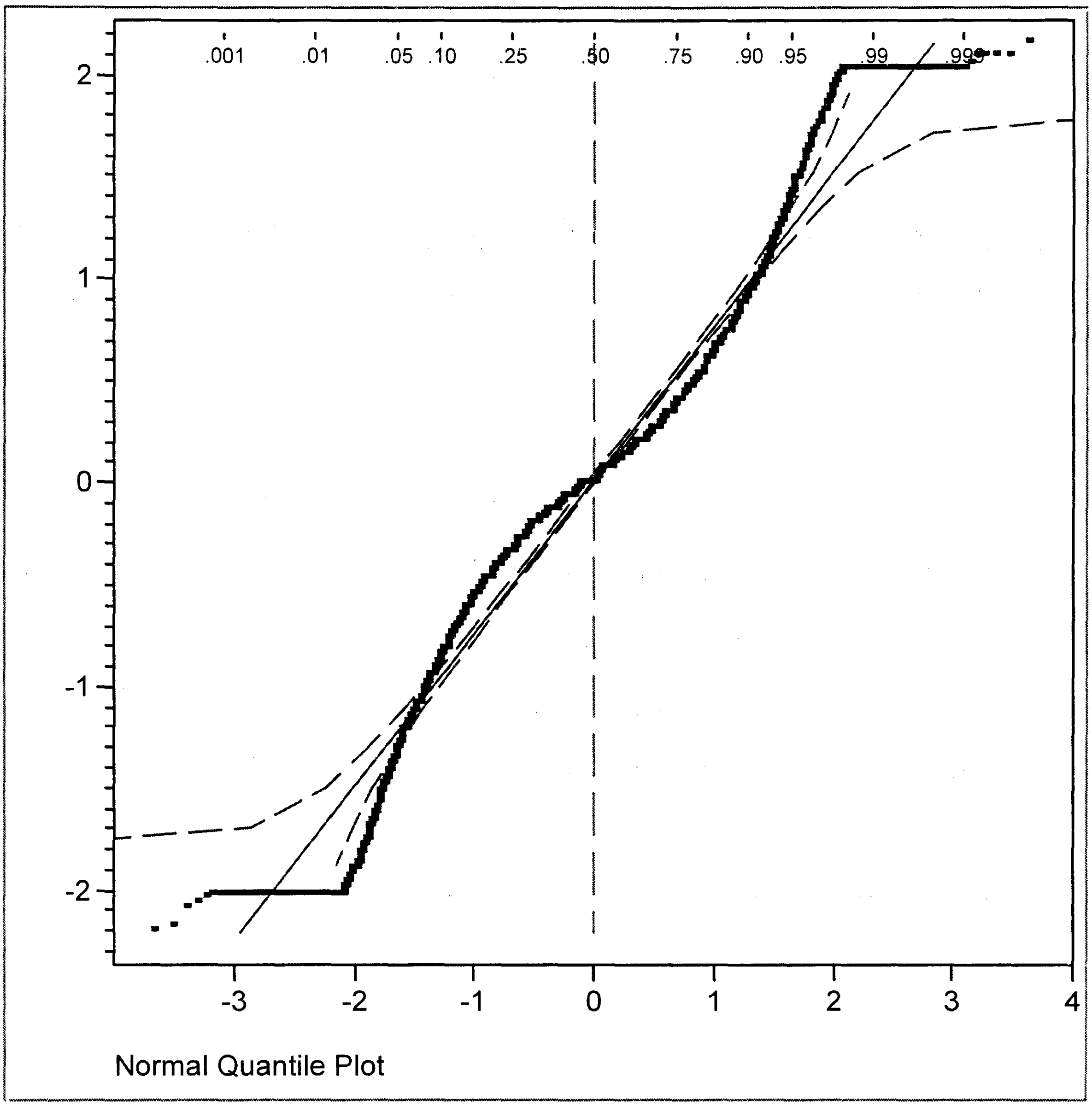

Figure 12. Normal probability plot for daily price changes on October contracts

Table 7. Statistics of the daily price changes on December contracts

Variable	N	Mean	Median	TrMean	StDev
DecPriceChange	8219	0.02013	0.00000	.02262	0.77296

Variable	SE Mean	Minimum	Maximum	Q1	Q3
DecPriceChange	0.00853	-5.60250	2.40300	-0.33750	0.40500

Observations = 8219

Sample Mean	=	0.02013365495	Variance	=	0.597462
Standard Error	=	0.77295694521	SE of Sample Mean	=	0.008526
t-Statistic	=	2.36144	Signif Level (Mean=0)	=	0.01822735
Skewness	=	-0.08652	Signif Level (Sk=0)	=	0.00136669
Kurtosis	=	1.30095	Signif Level (Ku=0)	=	0.00000000

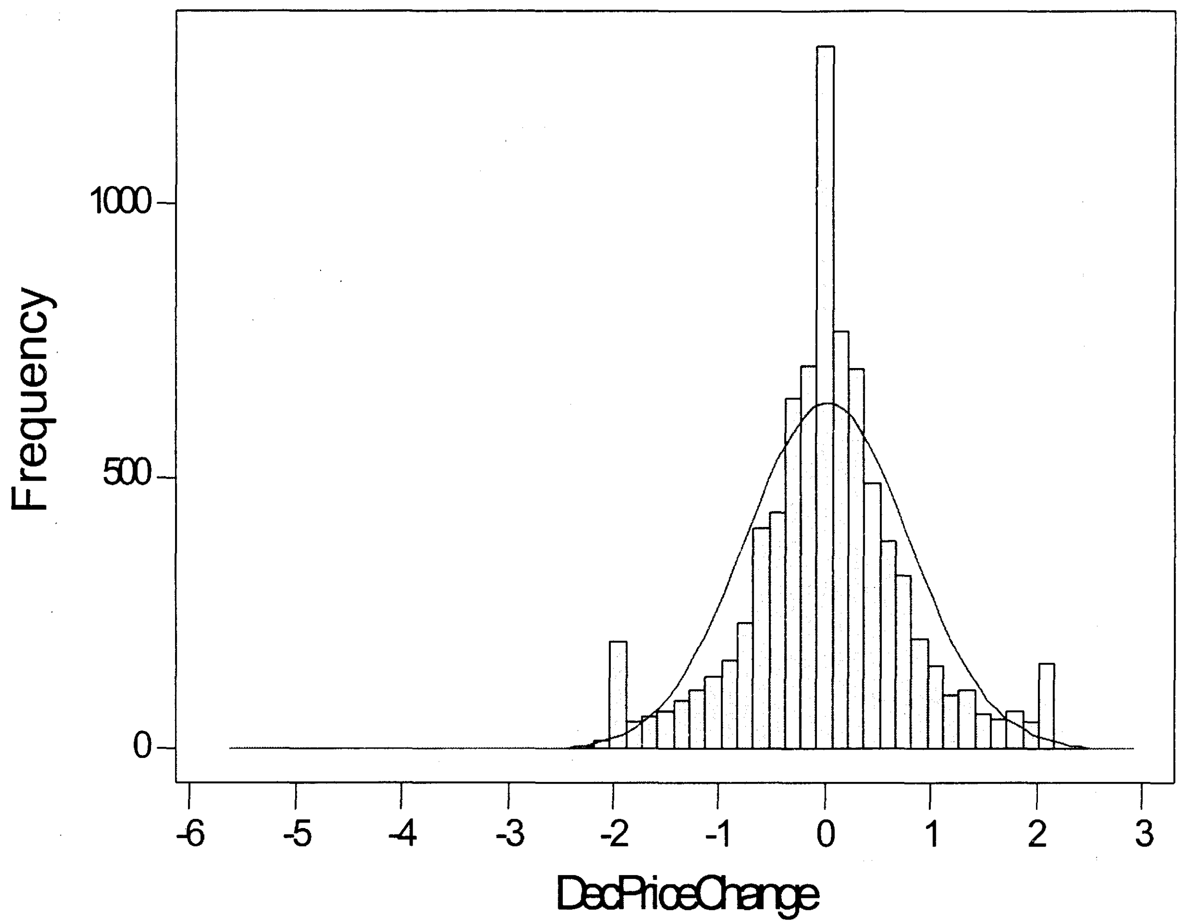

Figure 13. Histogram with normal curve for daily price changes on December contracts

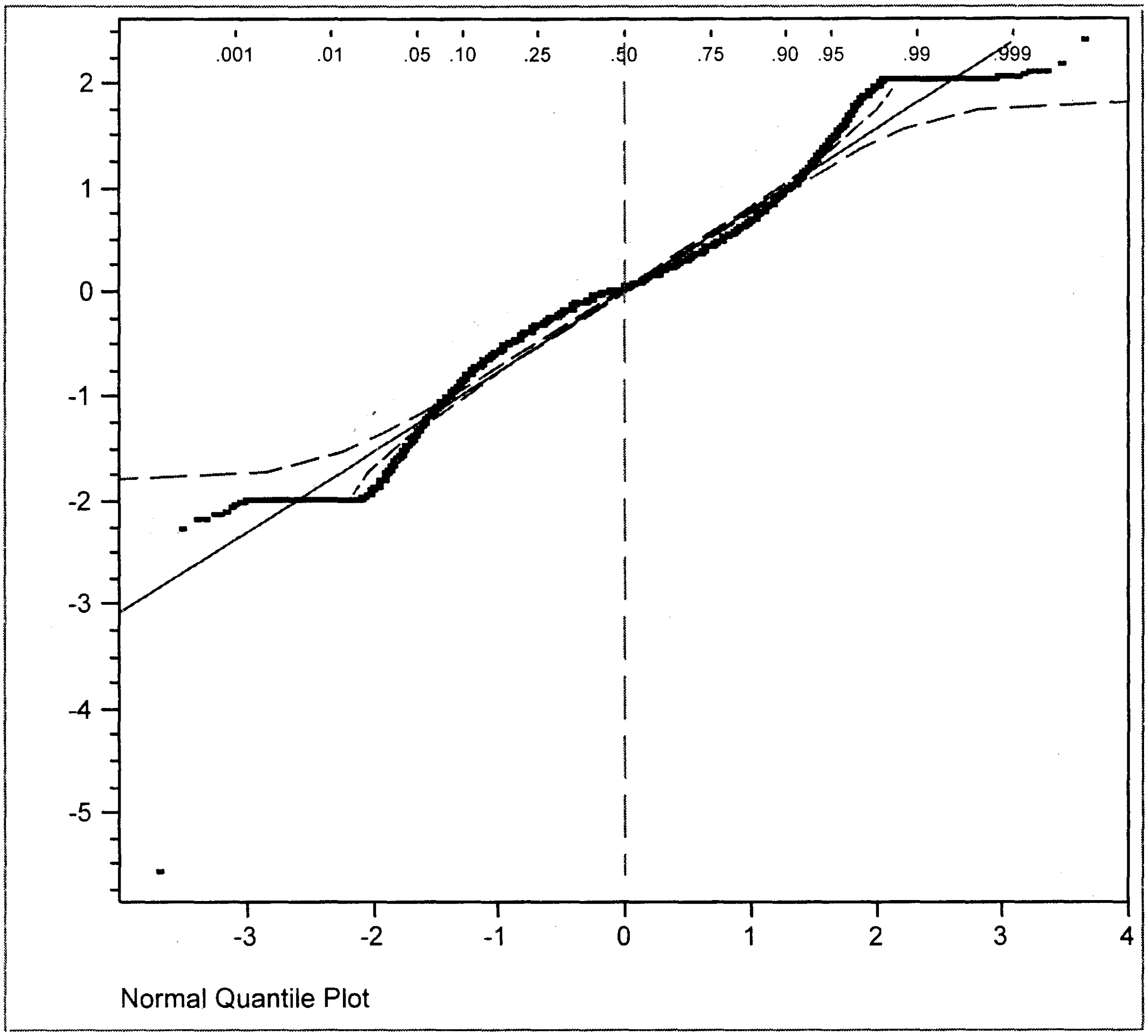

Figure 14. Normal probability plot for daily price changes on December contracts

Table 8. Statistics of the daily price changes on all contracts

Variable	N	Mean	Median	StDev
AllContracts	56227	0.0207568	0.0000	0.7720577

Variable	SE Mean	Minimum	Maximum	Q1	Q3
AllContracts	0.0032559	-5.6025	4.1850	-0.3375	0.4050

Observations = 56227

Sample Mean	=	0.02075684369	Variance	=	0.596073
Standard Error	=	0.77205774568	SE of Sample Mean	=	0.003256
t-Statistic	=	6.37506	Signif Level (Mean=0)	=	0.00000000
Skewness	=	-0.05202	Signif Level (Sk=0)	=	0.00000048
Kurtosis	=	1.06860	Signif Level (Ku=0)	=	0.00000000

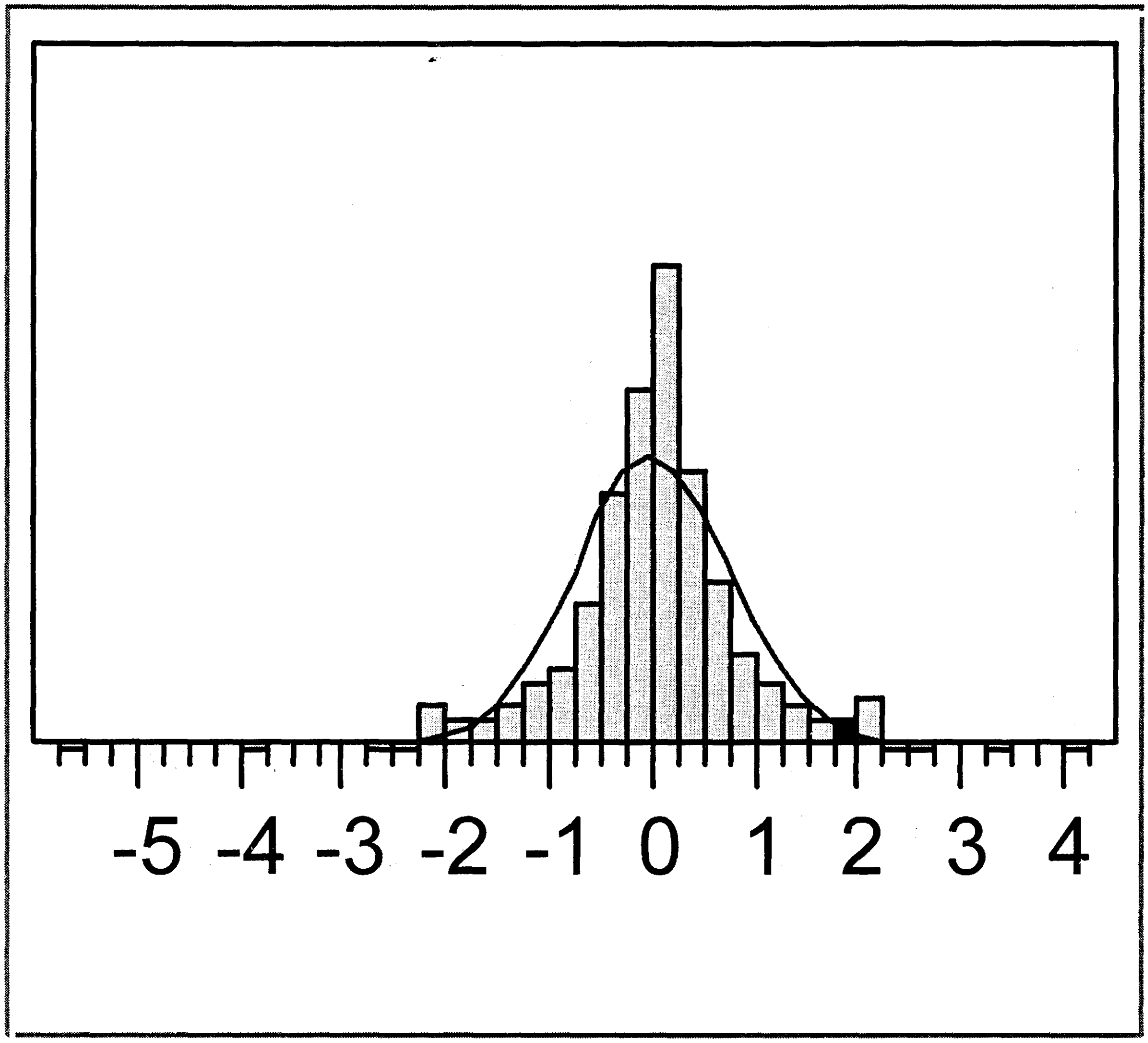

Figure 15. Histogram with normal curve for daily price changes on all contracts

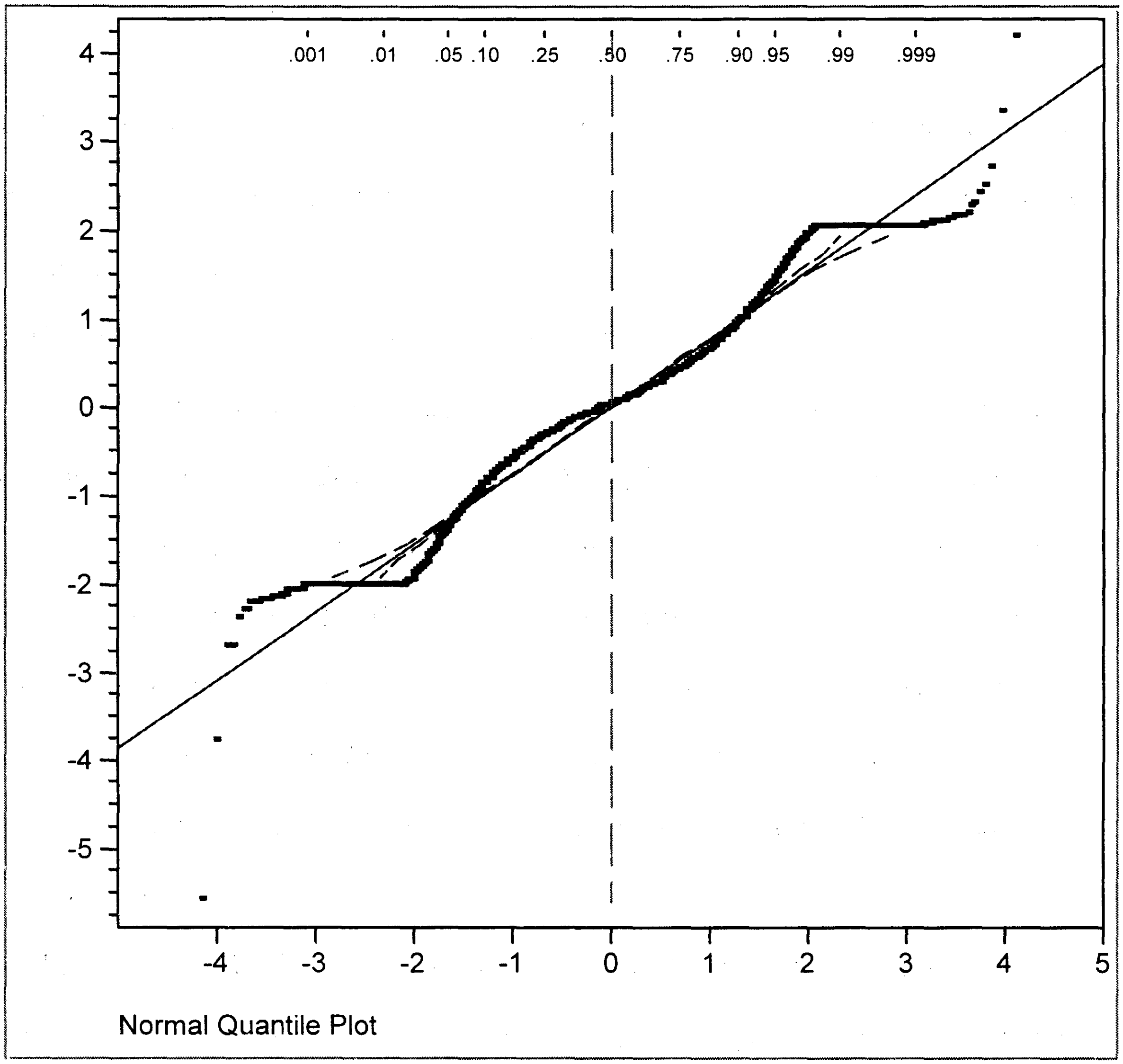

Figure 16. Normal probability plot for daily price changes on all contracts

The histograms for February, April, June, July, August, October, December and AllContracts contracts are given in Figure1, Figure 3, Figure 5, Figure 7, Figure 9, Figure 11, Figure 13 and Figure 15 respectively. All those histograms show that the distribution of the daily price changes has more mass in the tails and in the center than it should have under the assumption of normality.

The values of the skewness and the kurtosis and the significance level of their test statistic for February, April, June, July, August, October, December and AllContracts contracts are given in Table 1, Table 2, Table 3, Table 4, Table 5, Table 6, Table 7 and Table 8 respectively. The skewness is statistically not different from zero for June, July and October contracts but statistically different from zero for February, April, August, December and AllContracts contracts. Statistical softwares give the value of the kurtosis as the value of the kurtosis minus three. Thus the statistical softwares check for whether the kurtosis is statistically different from zero instead of three. The kurtosis is statistically different from three for all those contracts.

The normal probability plots for February, April, June, July, August, October, December and AllContracts contracts are given in Figure 2, Figure 4, Figure 6, Figure 8, Figure 10, Figure 12, Figure 14 and Figure 16 respectively. All those normal probability plots suggest that the distribution of daily price changes in the live hog futures is not normal.

The mean daily price changes and the significance level of their sample statistic for February, April, June, July, August, October, December and AllContracts contracts are given in Table 1, Table 2, Table 3, Table 4, Table 5, Table 6, Table 7 and Table 8 respectively. The mean is statistically not different from zero for April contracts but statistically different from zero for February, June, July, August, October, December and AllContracts contracts.

The sample variance of daily price changes for February, April, June, July, August, October, December and AllContracts contracts are 0.579581, 0.579399, 0.597178, 0.611788, 0.639417, 0.571054, 0.597462 and 0.596073 respectively. It is evident that the sample variability in the daily price changes is consistent.

Thus there appears to be significant evidence that the daily price changes in the live hog futures are not normally distributed and don't have zero mean either.

2.2 Reduced Data and Analysis

Since the price quotes, which are more than six months old, may not reflect the market situation very clearly, the data set was reduced to price changes dated within 6 months of the expiration of the contract. This reduced data set had 3582 observations of daily price changes for the February contracts, 3524 observations of daily price changes for the April contracts, 3528 observations of daily price changes for the June contracts, 3536 observations of daily price changes for the July contracts, 3427 observations of daily price changes for the August contracts, 3479 observations of daily price changes for the October contracts and 3473 observations of daily price changes for the December contracts. Altogether the data set had 24549 observations of daily price changes and it will be named as All-Contracts.

For all the observations on the February contracts, summary statistics have been presented in Table 9, the histogram with normal curve of this data has been presented in Figure 17 and the normal probability plot of this data has been presented in Figure 18. For all the observations on the April contracts, summary statistics have been presented in Table 10, histogram and normal probability plots are given in Figure 19 and Figure 20 respectively. For

Table 9. Statistics of the daily price changes on reduced February contracts

Variable	N	Mean	Median	TrMean	StDev
Feb6MosPrice	3582	0.0200	0.0270	0.0239	0.8395

Variable	SE Mean	Minimum	Maximum	Q1	Q3
Feb6MosPrice	0.0140	-2.2275	2.0925	-0.4320	0.4995

Observations = 3582

Sample Mean	=	0.01996509771	Variance	=	0.704792
Standard Error	=	0.83951905150	SE of Sample Mean	=	0.014027
t-Statistic	=	1.42332	Signif Level (Mean=0)	=	0.15472946
Skewness	=	-0.09392	Signif Level (Sk=0)	=	0.02179972
Kurtosis	=	0.44965	Signif Level (Ku=0)	=	0.00000004

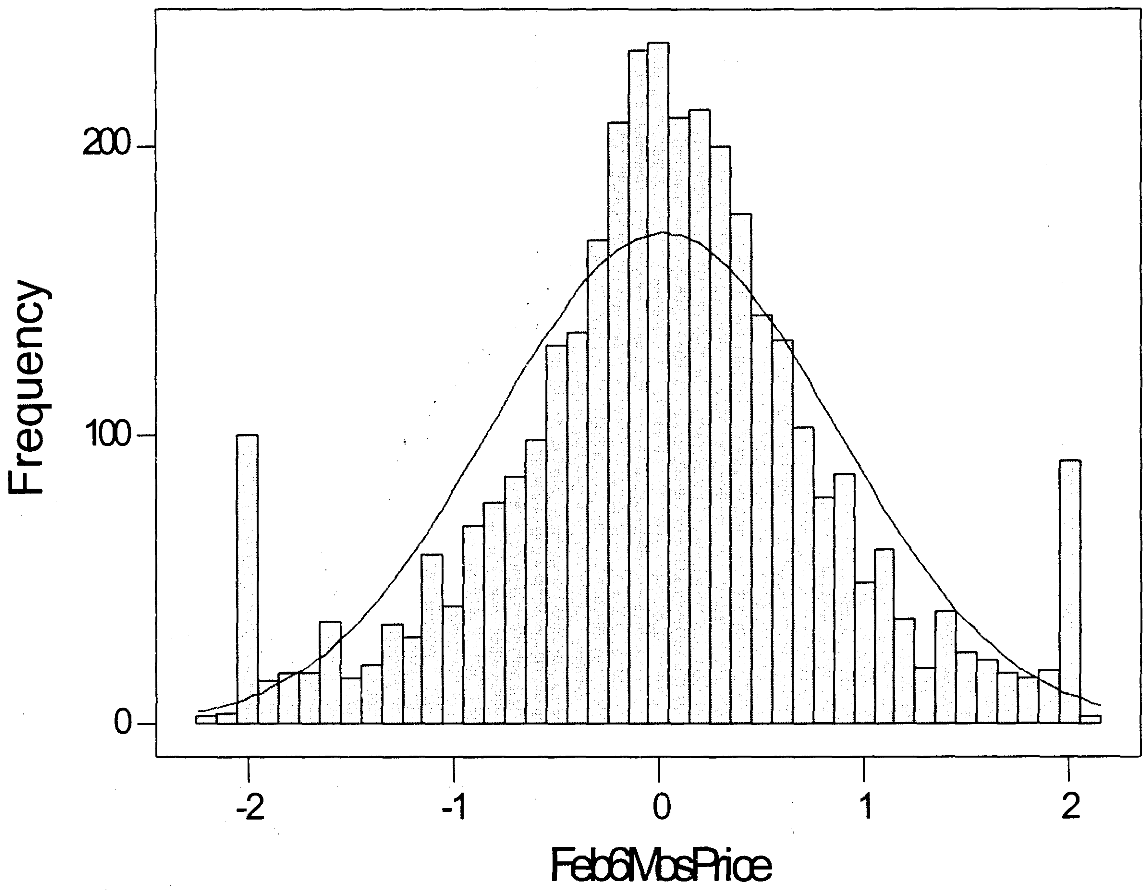

Figure 17. Histogram with normal curve for price changes on reduced February contracts

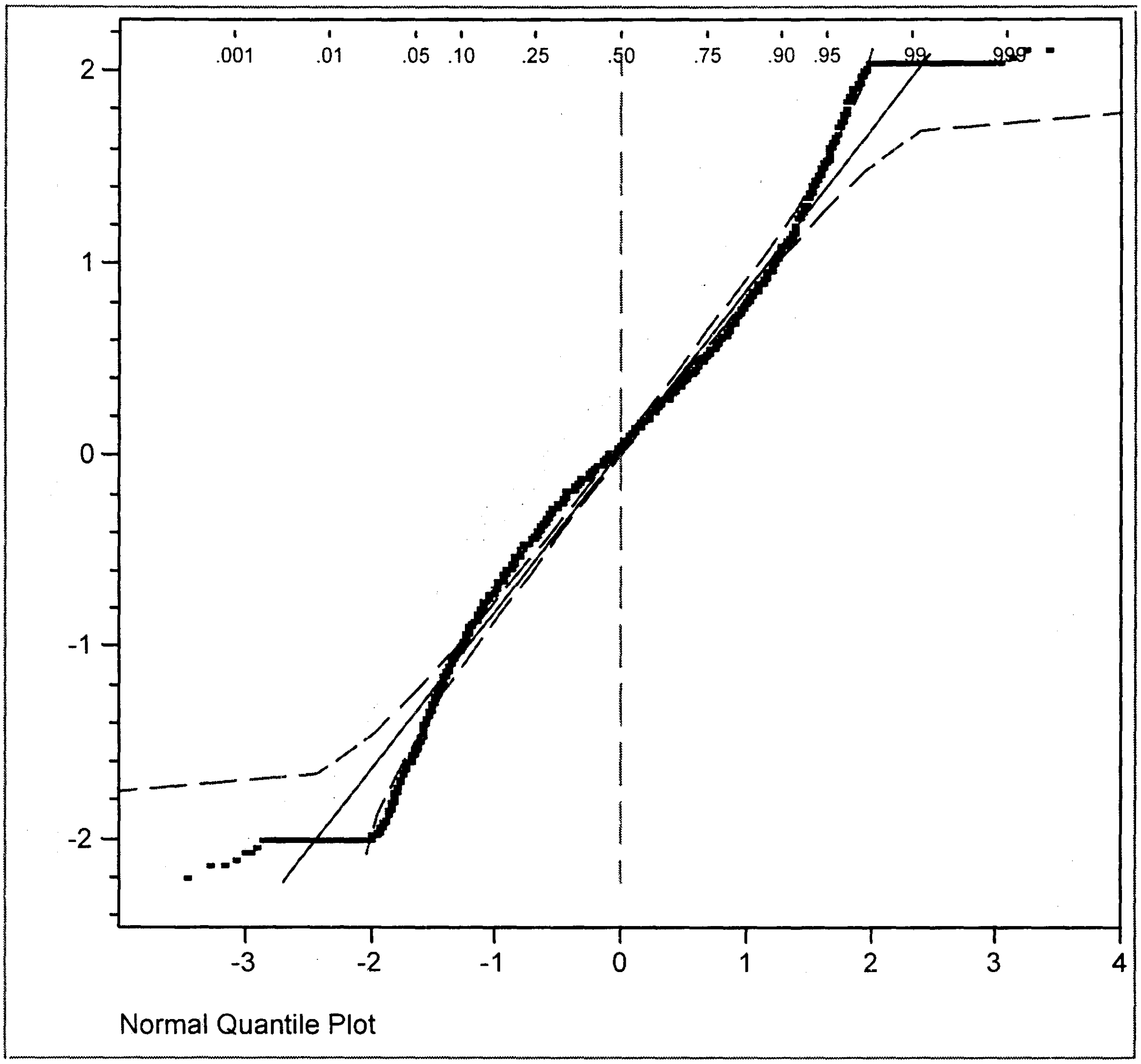

Figure 18. Normal probability plot for daily price changes on reduced February contracts

Table 10. Statistics of the daily price changes on reduced April contracts

Variable	N	Mean	Median	TrMean	StDev
Apr6MosPrice	3524	0.0189	0.0100	0.0234	0.7984

Variable	SE Mean	Minimum	Maximum	Q1	Q3
Apr6MosPrice	0.0134	-2.4030	2.2545	-0.4050	0.4725

Observations = 3524

Sample Mean	=	0.01888251986	Variance	=	0.637380
Standard Error	=	0.79836069309	SE of Sample Mean	=	0.013449
t-Statistic	=	1.40404	Signif Level (Mean=0)	=	0.16039578
Skewness	=	-0.07846	Signif Level (Sk=0)	=	0.05734642
Kurtosis	=	0.53240	Signif Level (Ku=0)	=	0.00000000

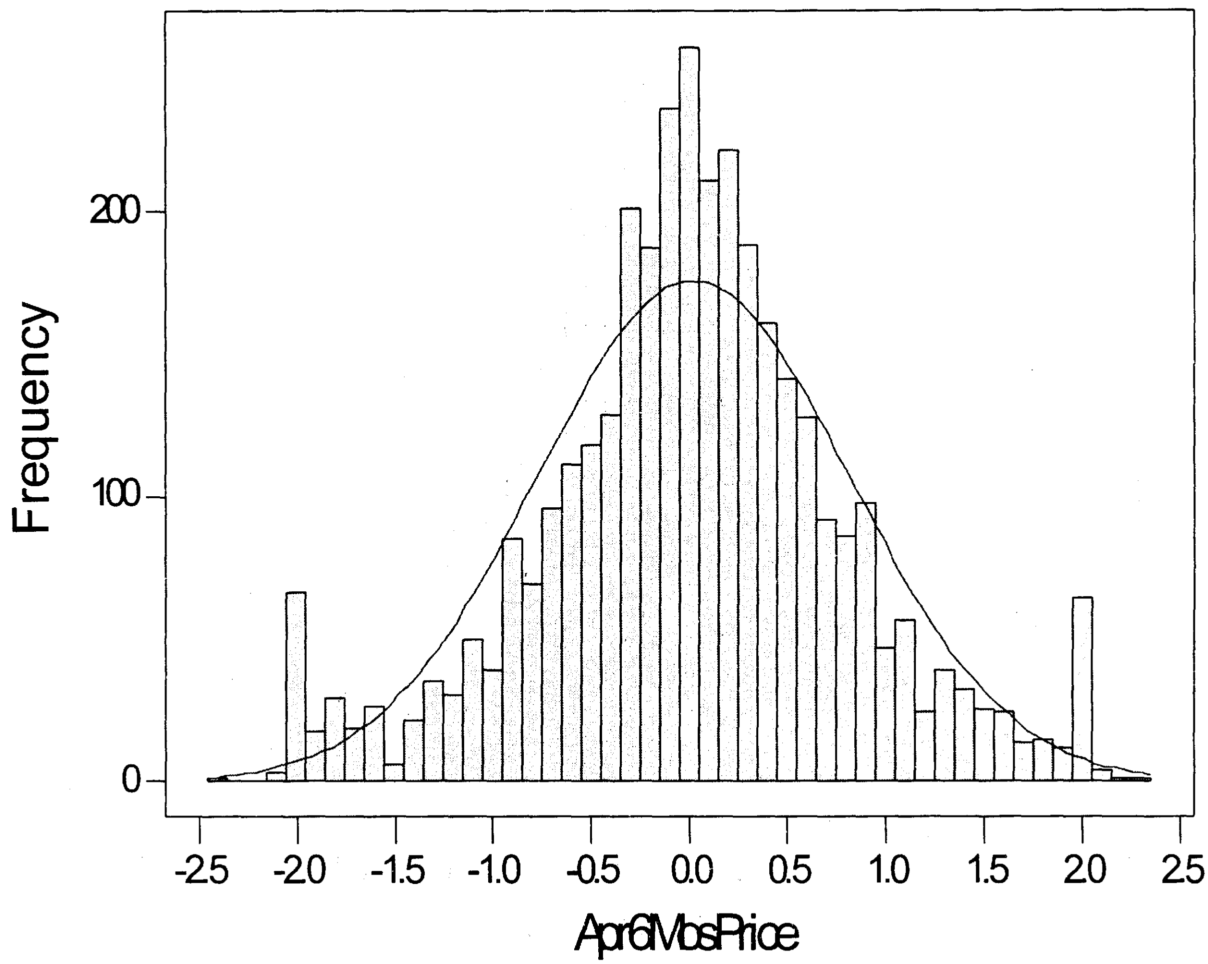

Figure 19. Histogram with normal curve for daily price changes on reduced April contracts

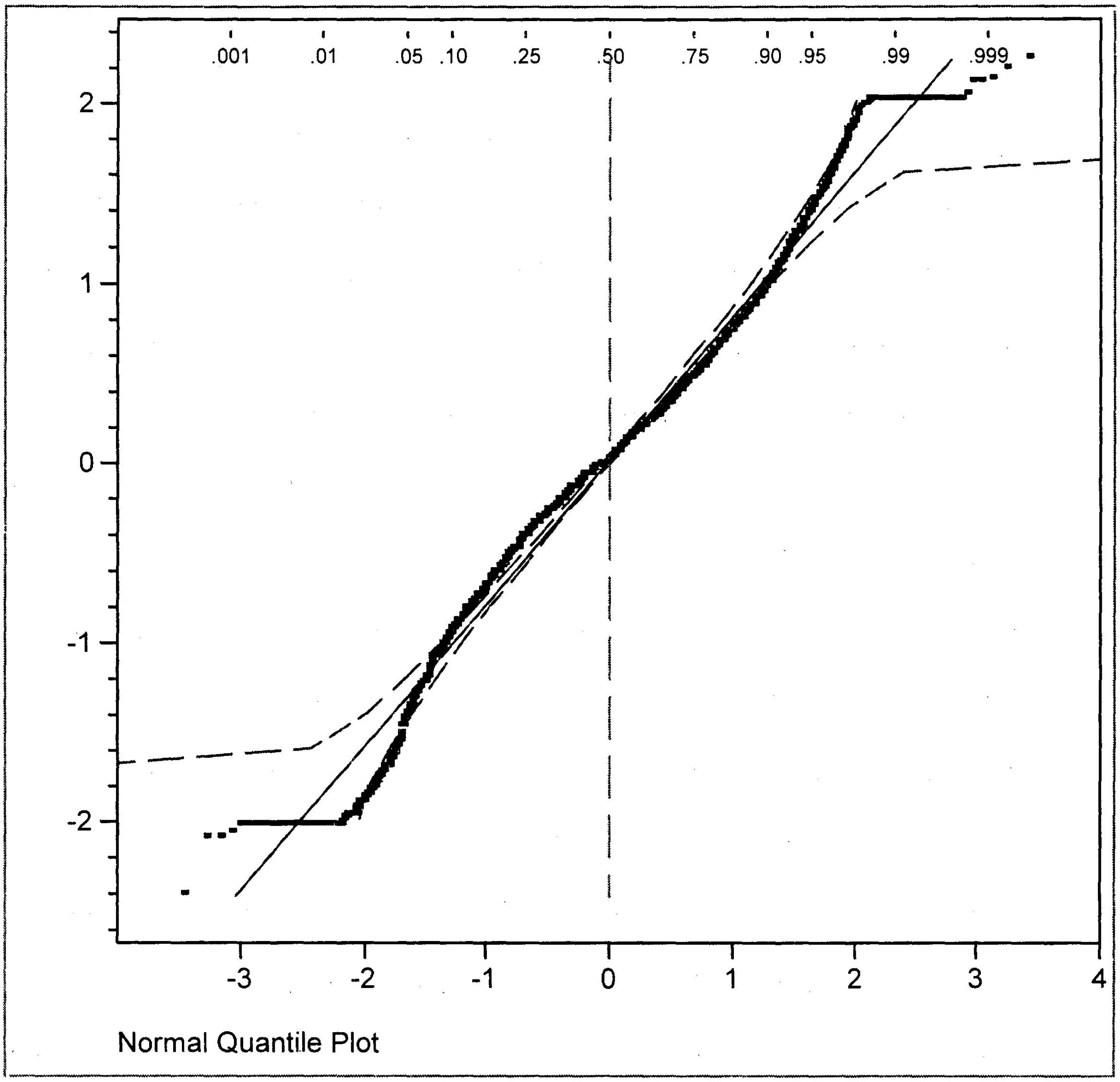

Figure 20. Normal probability plot for daily price changes on reduced April contracts

all the observations on the June contracts, summary statistics have been presented in Table 11, the histogram with normal curve of this data has been presented in Figure 21 and the normal probability plot of this data has been presented in Figure 22. For all the observations on the July contracts, summary statistics have been presented in Table 12, the histogram with normal curve of this data has been presented in Figure 23 and the normal probability plot of this data has been presented in Figure 24. For all the observations on the August contracts, summary statistics have been presented in Table 13, the histogram with normal curve of this data has been presented in Figure 25 and the normal probability plot of this data has been presented in Figure 26. For all the observations on the October contracts, summary statistics have been presented in Table 14, the histogram with normal curve of this data has been presented in Figure 27 and the normal probability plot of this data has been presented in Figure 28. For all the observations on the December contracts, summary statistics have been presented in Table 15, the histogram with normal curve of this data has been presented in Figure 29 and the normal probability plot of this data has been presented in Figure 30. For all the observations on All-Contracts, summary statistics have been presented in Table 16, the histogram with normal curve of this data has been presented in Figure 31 and the normal probability plot of this data has been presented in Figure 32.

The histograms for February, April, June, July, August, October, December and All-Contracts contracts are given in Figure 17, Figure 19, Figure 21, Figure 23, Figure 25, Figure 27, Figure 29 and Figure 31 respectively. All those histograms show that the distribution of the daily price changes has more mass in the tails and in the center than it should have under the assumption of normality.

Table 11. Statistics of the daily price changes on reduced June contracts

Variable	N	Mean	Median	TrMean	StDev
Jun6MosPrice	3528	0.0463	0.0405	0.0486	0.8467

Variable	SE Mean	Minimum	Maximum	Q1	Q3
Jun6MosPrice	0.0143	-2.1330	2.1195	-0.4253	0.5183

Observations = 3528

Sample Mean	=	0.04633730442	Variance	=	0.716947
Standard Error	=	0.84672742913	SE of Sample Mean	=	0.014255
t-Statistic	=	3.25051	Signif Level (Mean=0)	=	0.00116281
Skewness	=	-0.01930	Signif Level (Sk=0)	=	0.63999697
Kurtosis	=	0.31474	Signif Level (Ku=0)	=	0.00013773

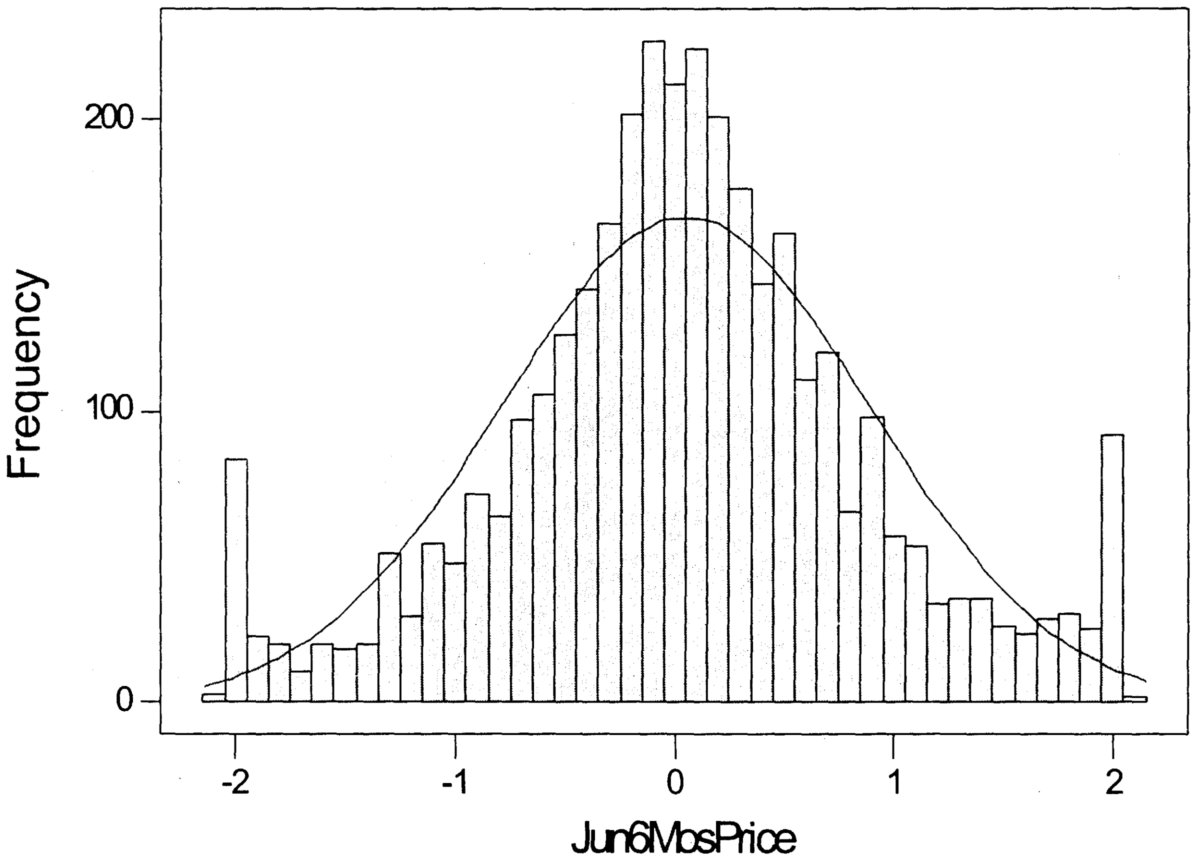

Figure 21. Histogram with normal curve for daily price changes on reduced June contracts

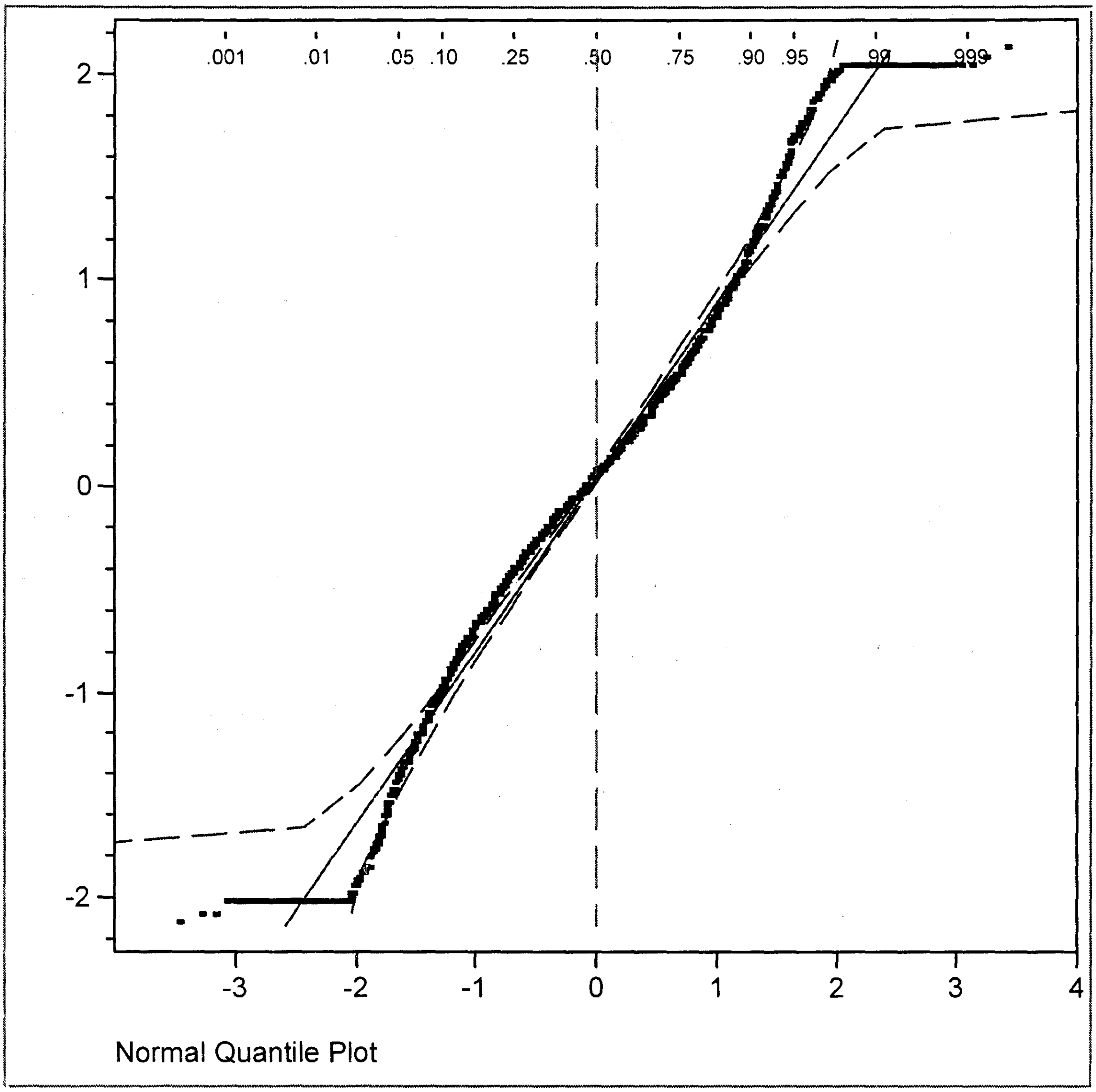

Figure 22. Normal probability plot for daily price changes on reduced June contracts

Table 12. Statistics of the daily price changes on reduced July contracts

Variable	N	Mean	Median	TrMean	StDev
Jul6MosPrice	3536	0.0340	0.0405	0.0376	0.8458

Variable	SE Mean	Minimum	Maximum	Q1	Q3
Jul6MosPrice	0.0142	-2.0250	2.0925	-0.4320	0.5130

Observations = 3536

Sample Mean	=	0.03402347002	Variance	=	0.715327
Standard Error	=	0.84576997901	SE of Sample Mean	=	0.014223
t-Statistic	=	2.39212	Signif Level (Mean=0)	=	0.01680342
Skewness	=	-0.08019	Signif Level (Sk=0)	=	0.05166448
Kurtosis	=	0.31702	Signif Level (Ku=0)	=	0.00012097

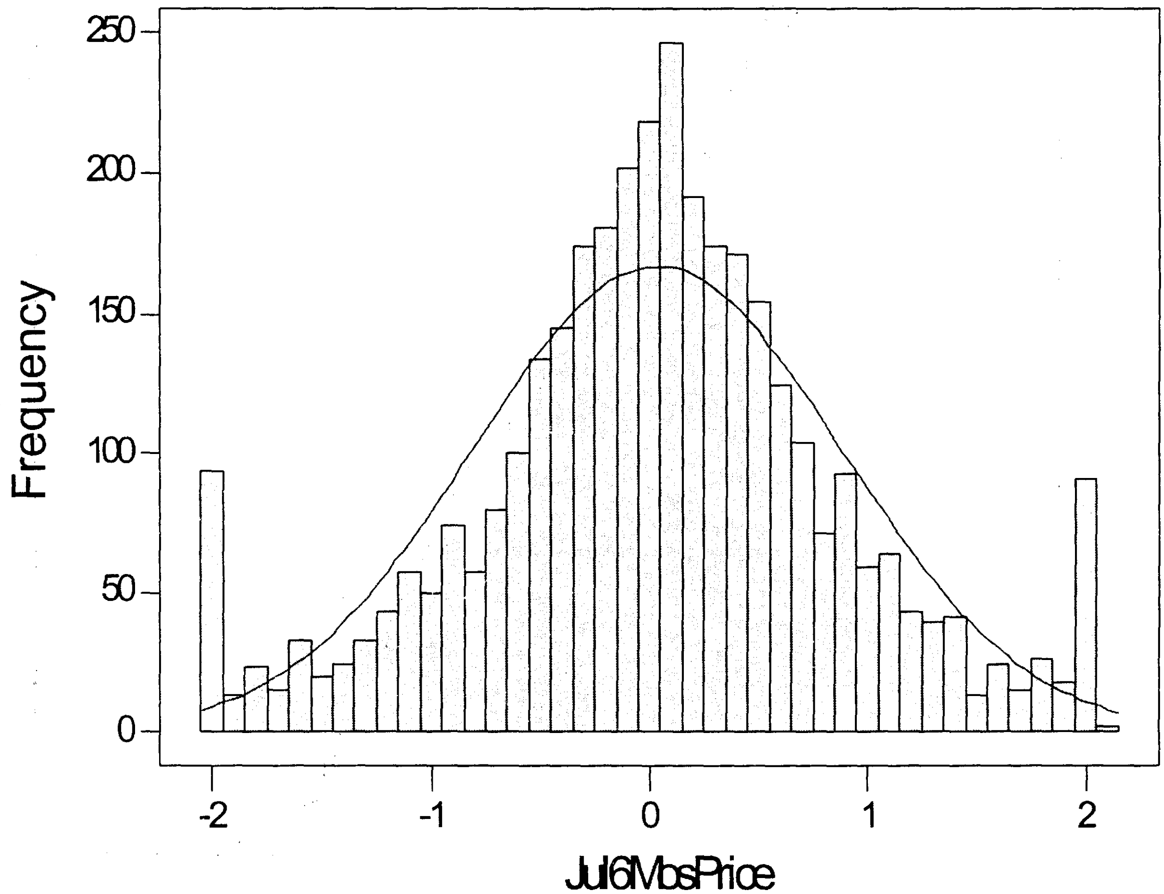

Figure 23. Histogram with normal curve for daily price changes on reduced July contracts

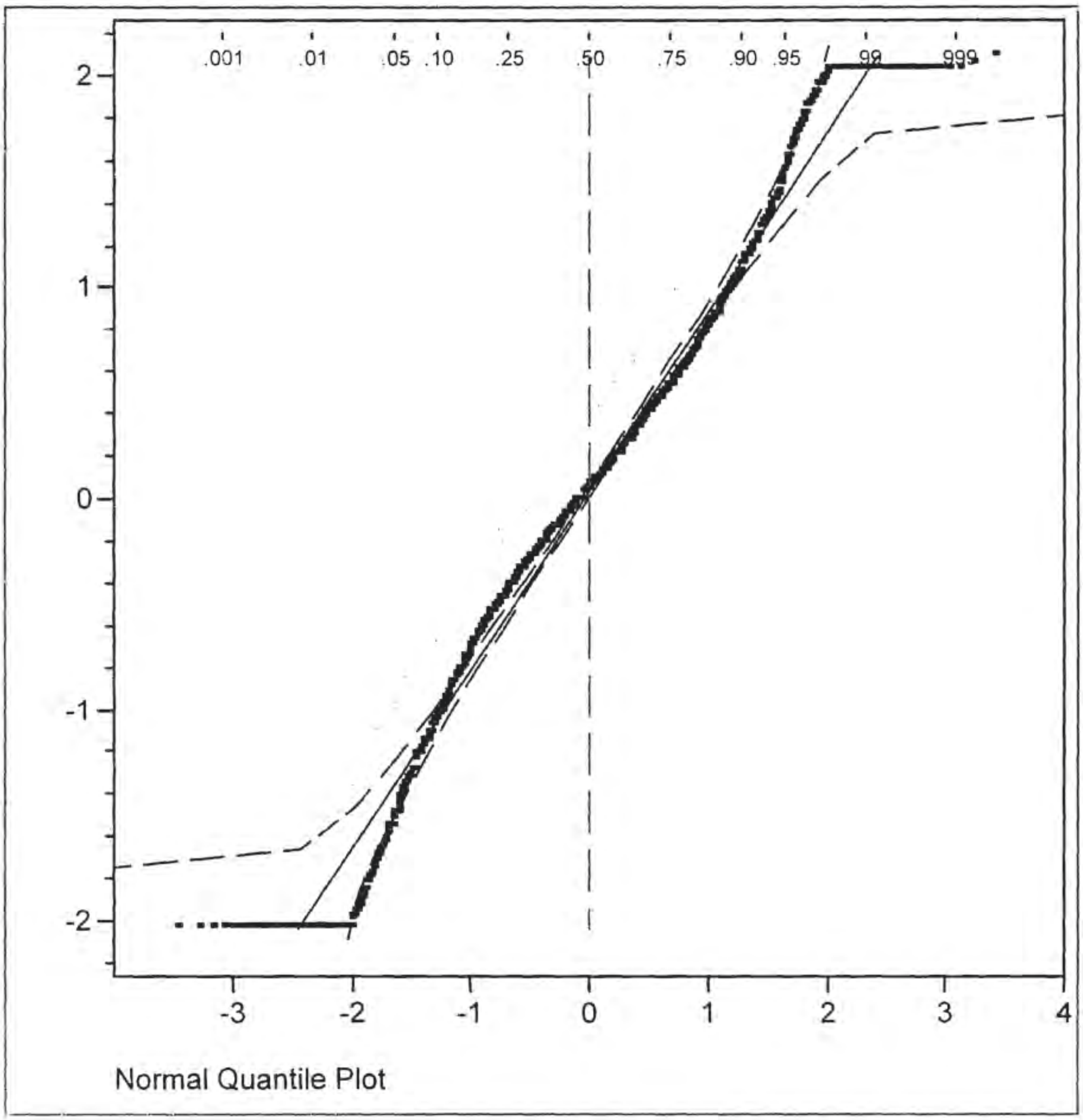

Figure 24. Normal probability plot for daily price changes on reduced July contracts

Table 13. Statistics of the daily price changes on reduced August contracts

Variable	N	Mean	Median	TrMean	StDev
Aug6MosPrice	3427	0.0443	0.0405	0.0495	0.8775

Variable	SE Mean	Minimum	Maximum	Q1	Q3
Aug6MosPrice	0.0150	-2.2005	2.1600	-0.4320	0.5400

Observations = 3427

Sample Mean	=	0.04434082580	Variance	=	0.769993
Standard Error	=	0.87749258413	SE of Sample Mean	=	0.014989
t-Statistic	=	2.95813	Signif Level (Mean=0)	=	0.00311628
Skewness	=	-0.10731	Signif Level (Sk=0)	=	0.01036370
Kurtosis	=	0.25004	Signif Level (Ku=0)	=	0.00283706

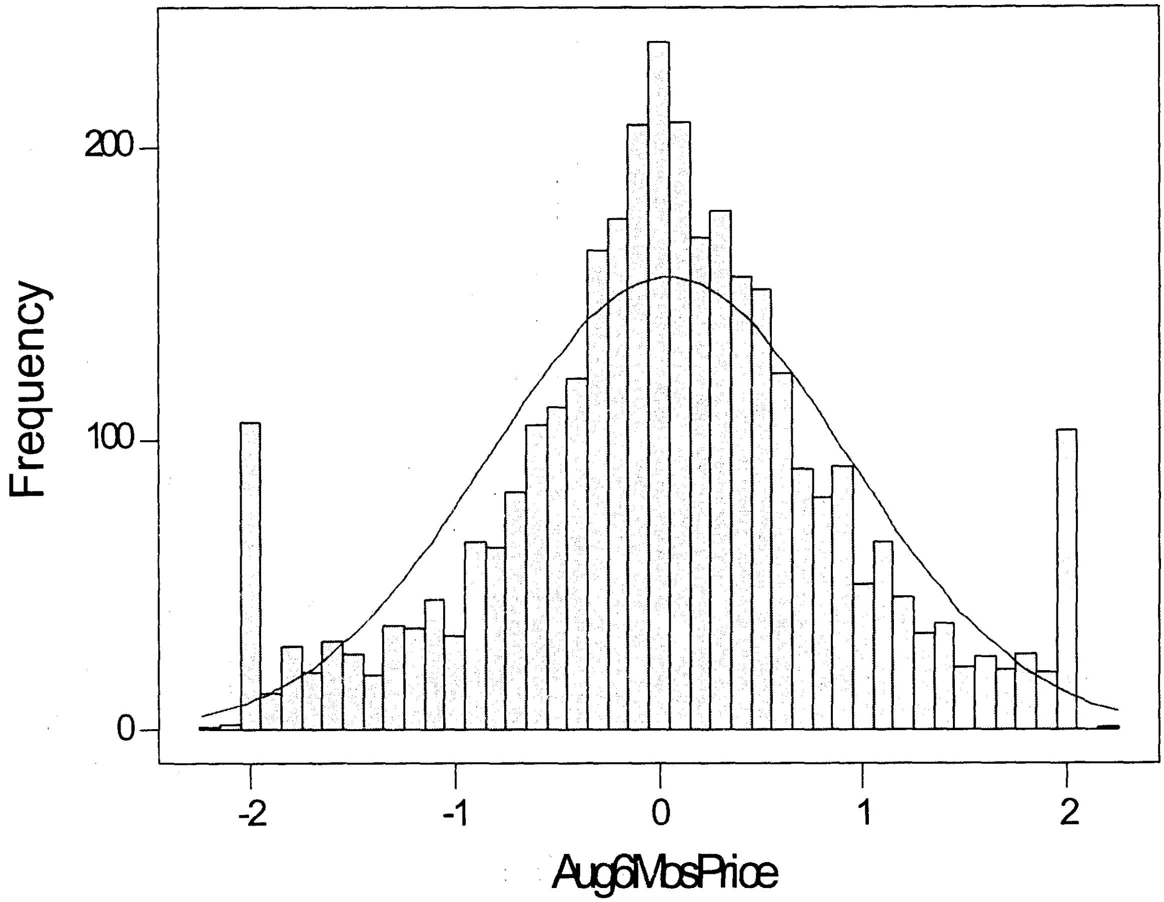

Figure 25. Histogram with normal curve for daily price changes on reduced August contracts

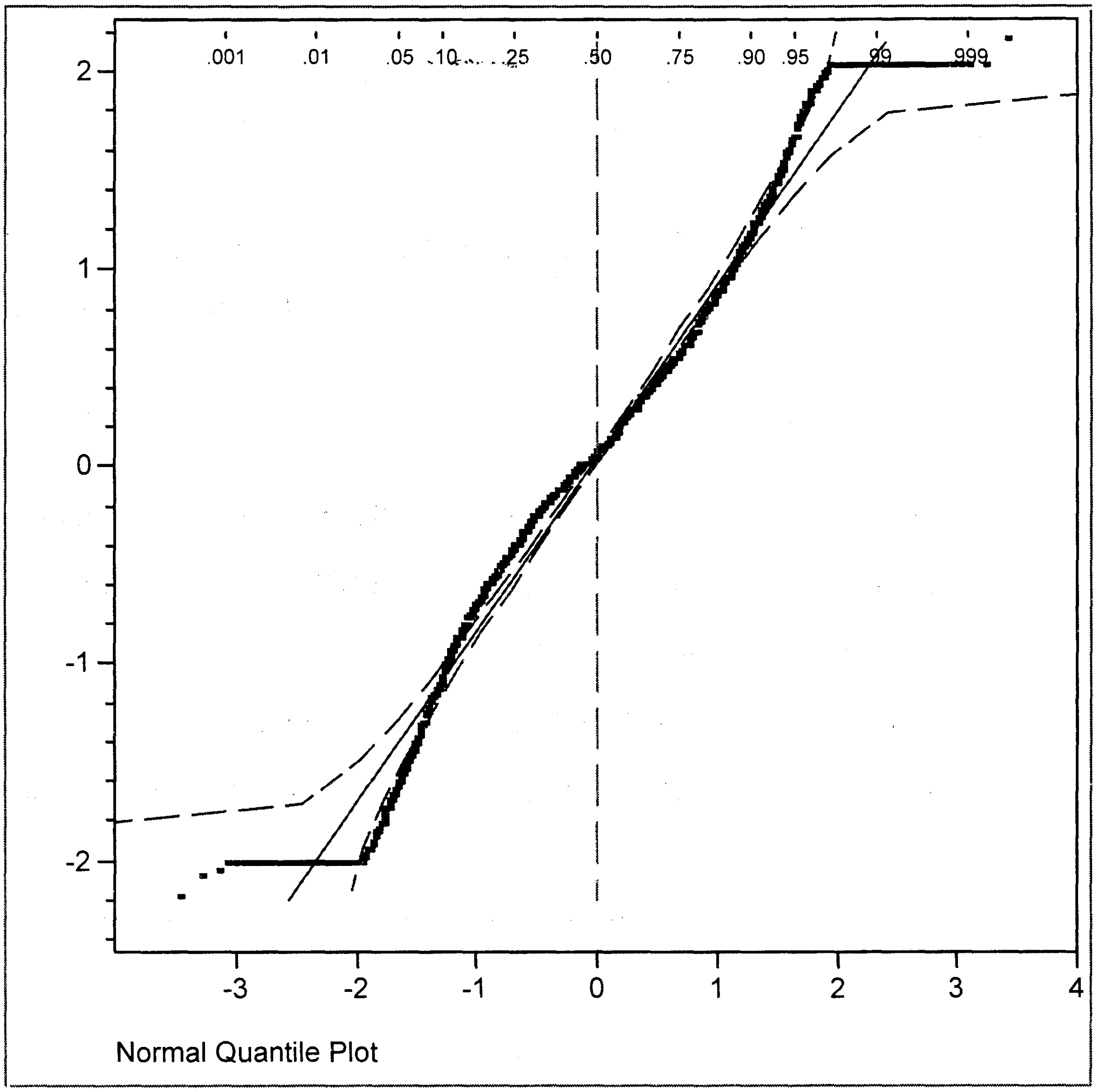

Figure 26. Normal probability plot for daily price changes on reduced August contracts

Table 14. Statistics of the daily price changes on reduced October contracts

Variable	N	Mean	Median	TrMean	StDev
Oct6MosPrice	3479	0.0170	0.0000	0.0203	0.8704

Variable	SE Mean	Minimum	Maximum	Q1	Q3
Oct6MosPrice	0.0148	-2.2005	2.0925	-0.4455	0.4995

Observations = 3479

Sample Mean	=	0.01701954010	Variance	=	0.757642
Standard Error	=	0.87042631010	SE of Sample Mean	=	0.014757
t-Statistic	=	1.15330	Signif Level (Mean=0)	=	0.24886572
Skewness	=	-0.05368	Signif Level (Sk=0)	=	0.19632334
Kurtosis	=	0.30090	Signif Level (Ku=0)	=	0.00029557

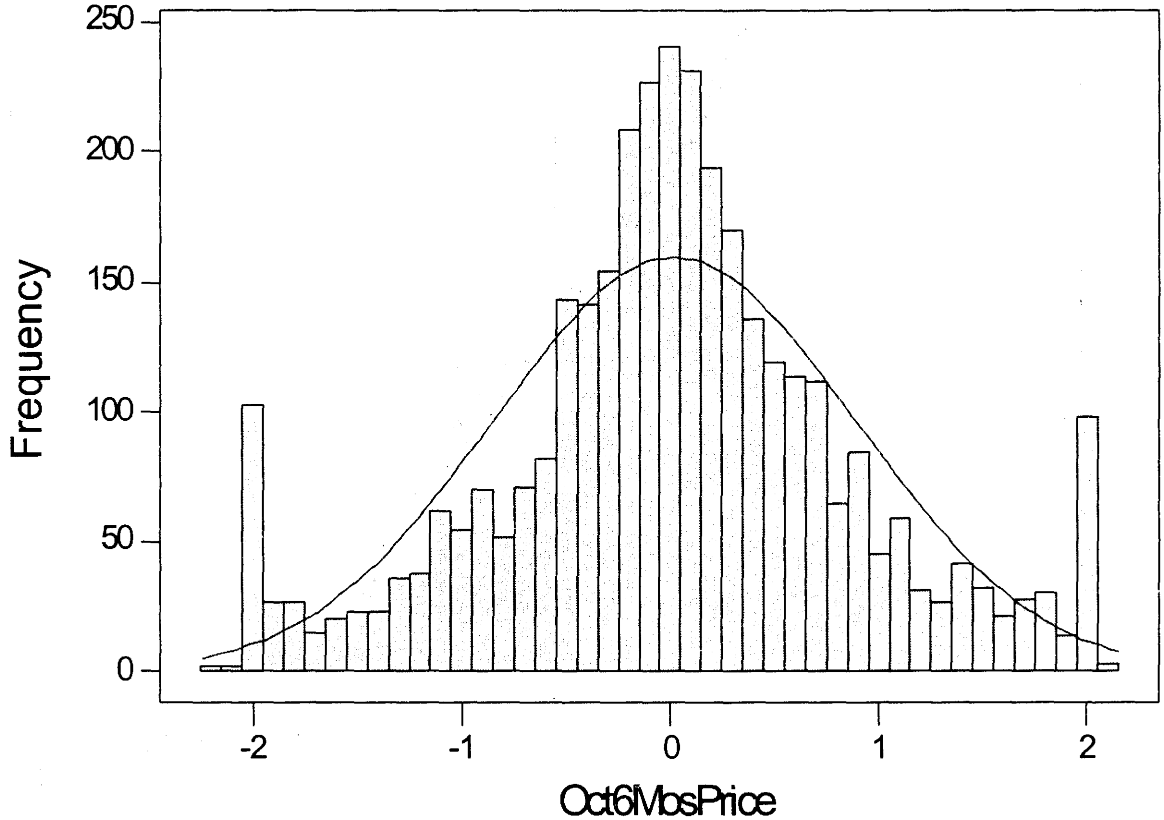

Figure 27. Histogram with normal curve for price changes on reduced October contracts

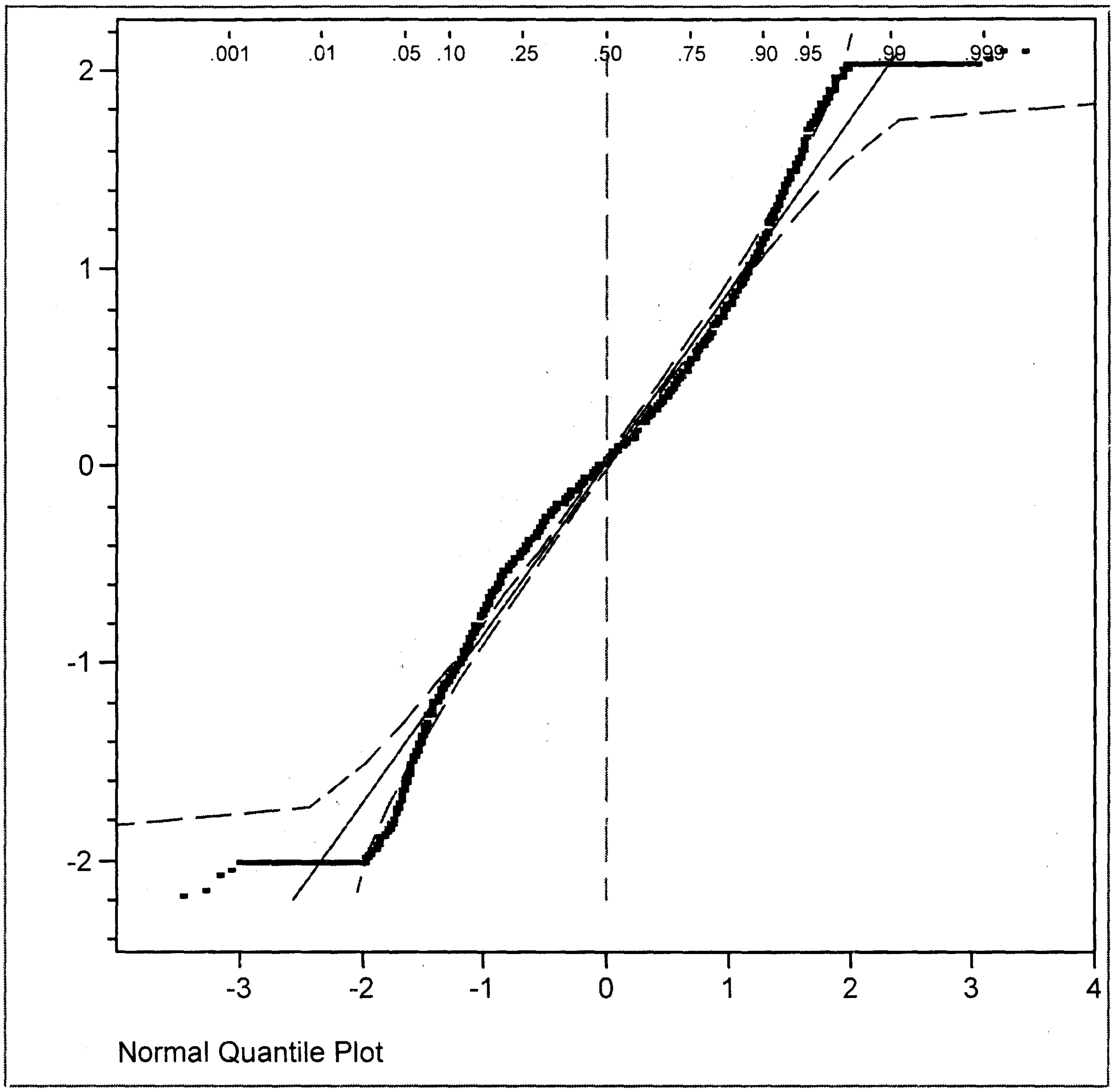

Figure 28. Normal probability plot for daily price changes on reduced October contracts

Table 15. Statistics of the daily price changes on reduced December contracts

Variable	N	Mean	Median	TrMean	StDev
Dec6MosPrice	3473	0.0253	0.0270	0.0280	0.9035

Variable	SE Mean	Minimum	Maximum	Q1	Q3
Dec6MosPrice	0.0153	-2.2005	2.4030	-0.4725	0.5400

Observations = 3473

Sample Mean	=	0.02530881371	Variance	=	0.816322
Standard Error	=	0.90350561339	SE of Sample Mean	=	0.015331
t-Statistic	=	1.65080	Signif Level (Mean=0)	=	0.09887074
Skewness	=	-0.02643	Signif Level (Sk=0)	=	0.52495896
Kurtosis	=	0.14301	Signif Level (Ku=0)	=	0.08568105

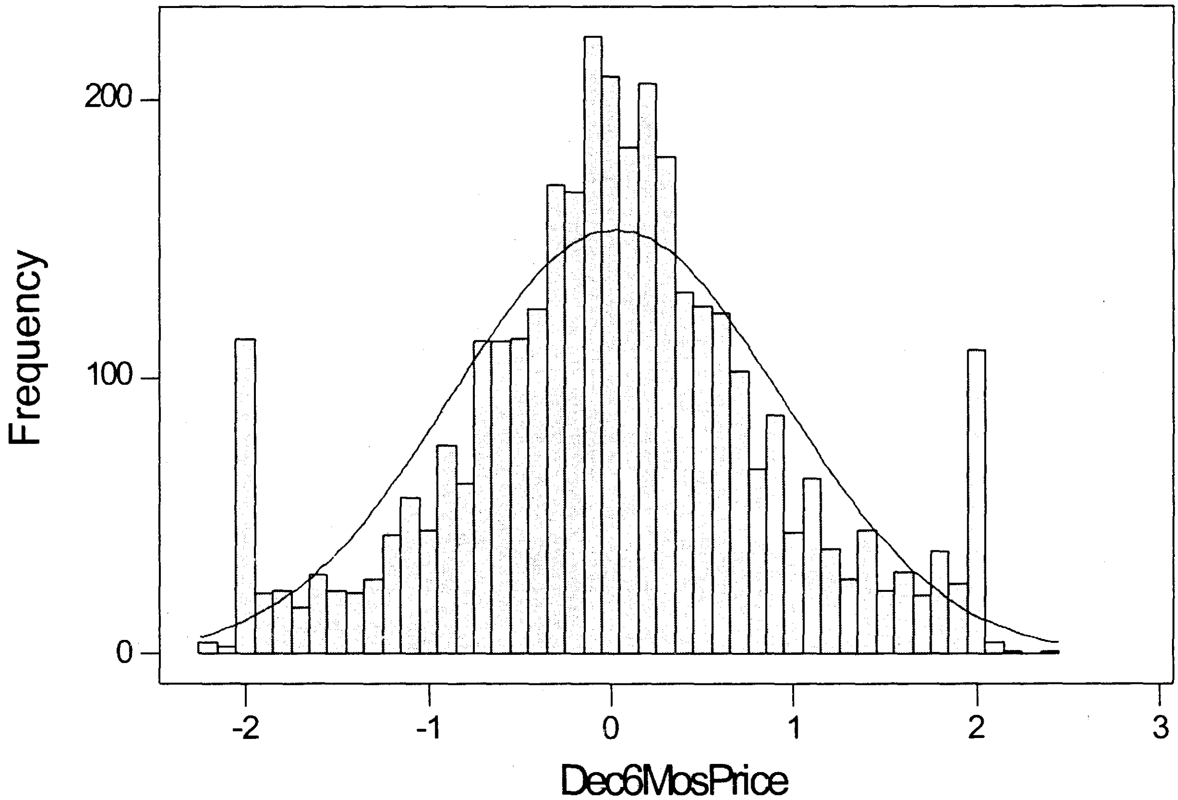

Figure 29. Histogram with normal curve for price changes on reduced December contracts

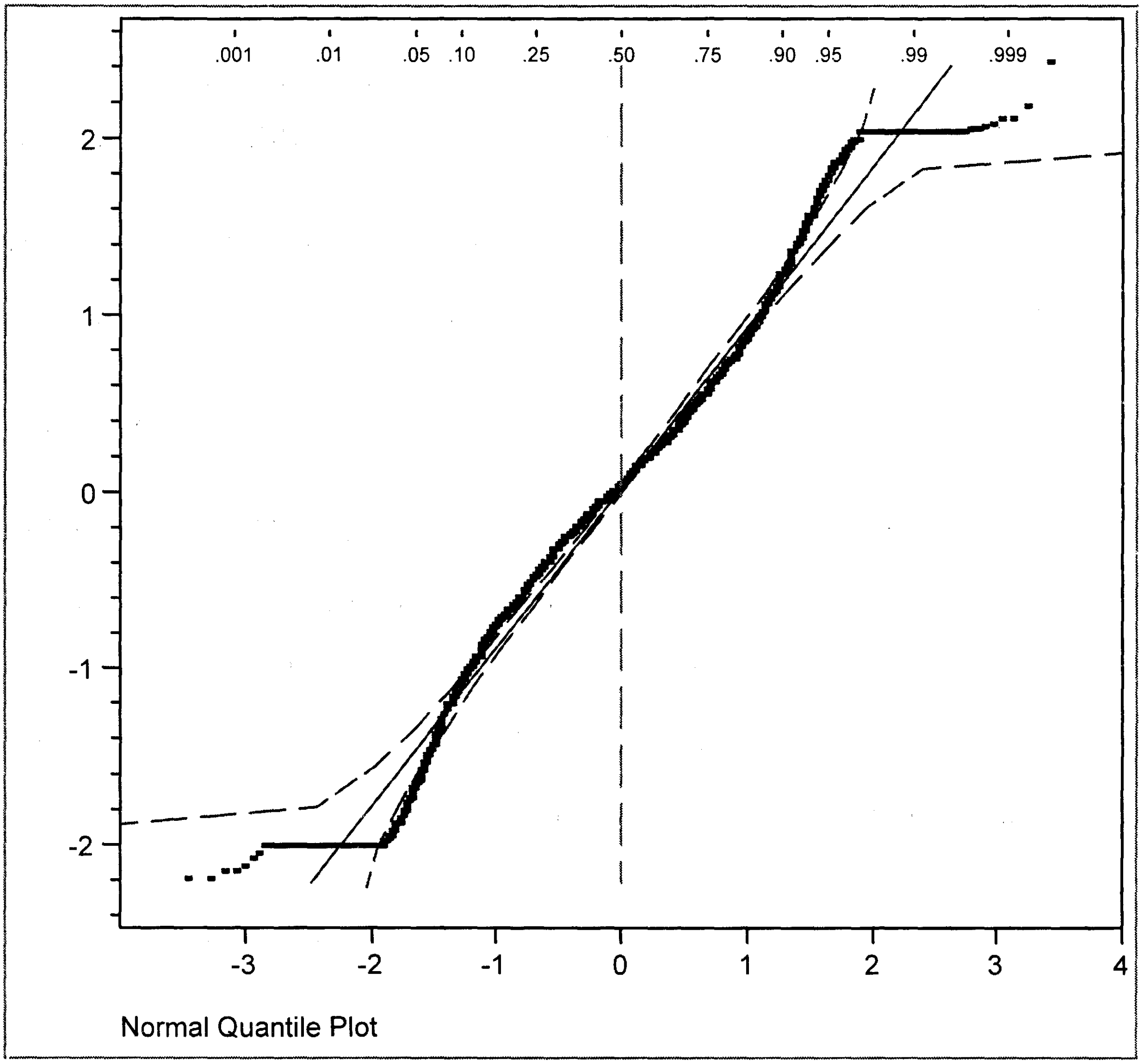

Figure 30. Normal probability plot for daily price changes on reduced December contracts

Table 16. Statistics of the daily price changes on reduced all contracts

Variable	N	Mean	Median	TrMean	StDev
All6MosPrice	24549	0.02937	0.02700	0.03282	0.85481

Variable	SE Mean	Minimum	Maximum	Q1	Q3
All6MosPrice	0.00546	-2.40300	2.40300	-0.43200	0.51300

Observations = 24549

Sample Mean	=	0.02936602224	Variance	=	0.730699
Standard Error	=	0.85480943783	SE of Sample Mean	=	0.005456
t-Statistic	=	5.38261	Signif Level (Mean=0)	=	0.00000007
Skewness	=	-0.06408	Signif Level (Sk=0)	=	0.00004158
Kurtosis	=	0.32984	Signif Level (Ku=0)	=	0.00000000

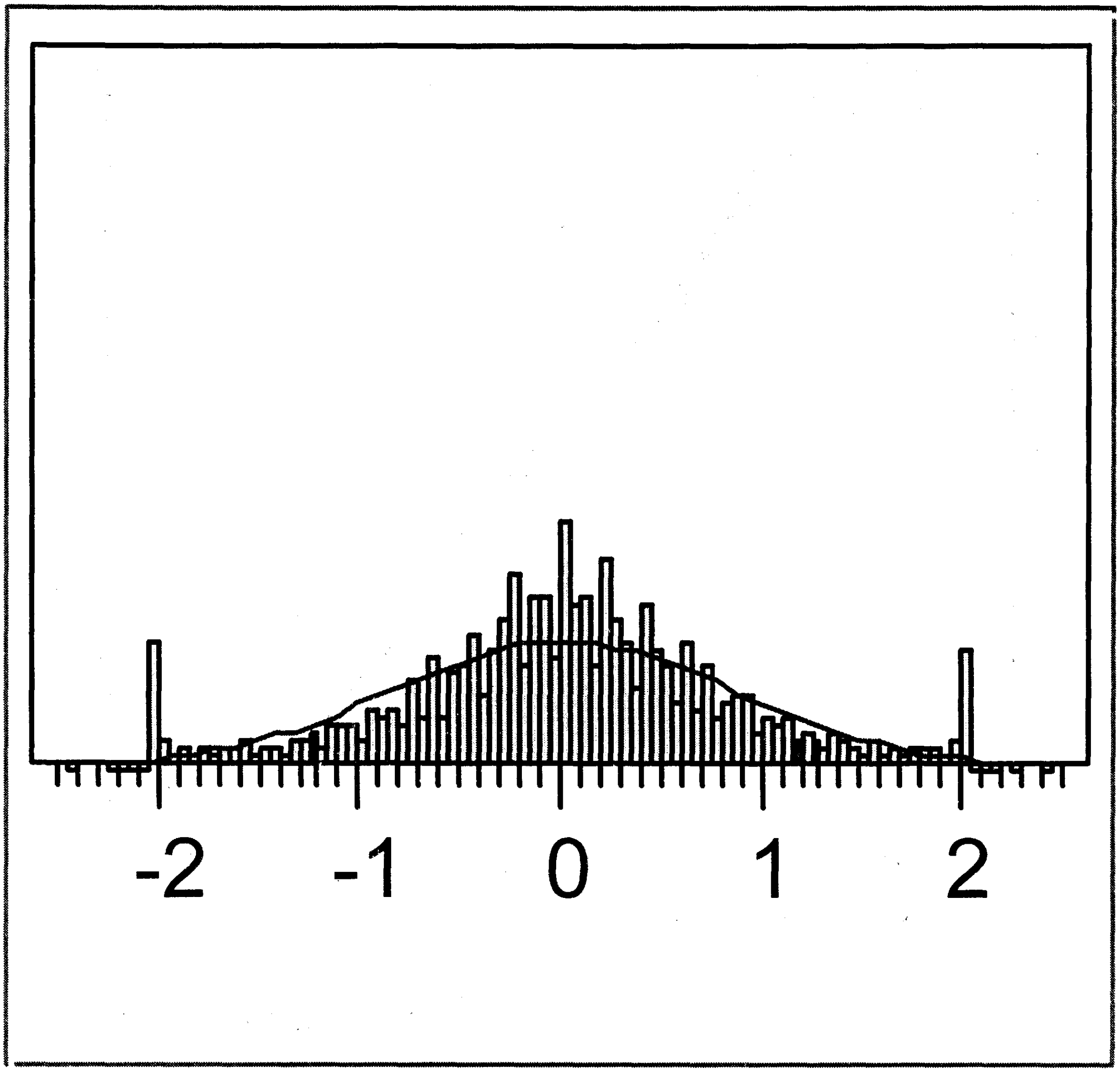

Figure 31. Histogram with normal curve for daily price changes on reduced all contracts

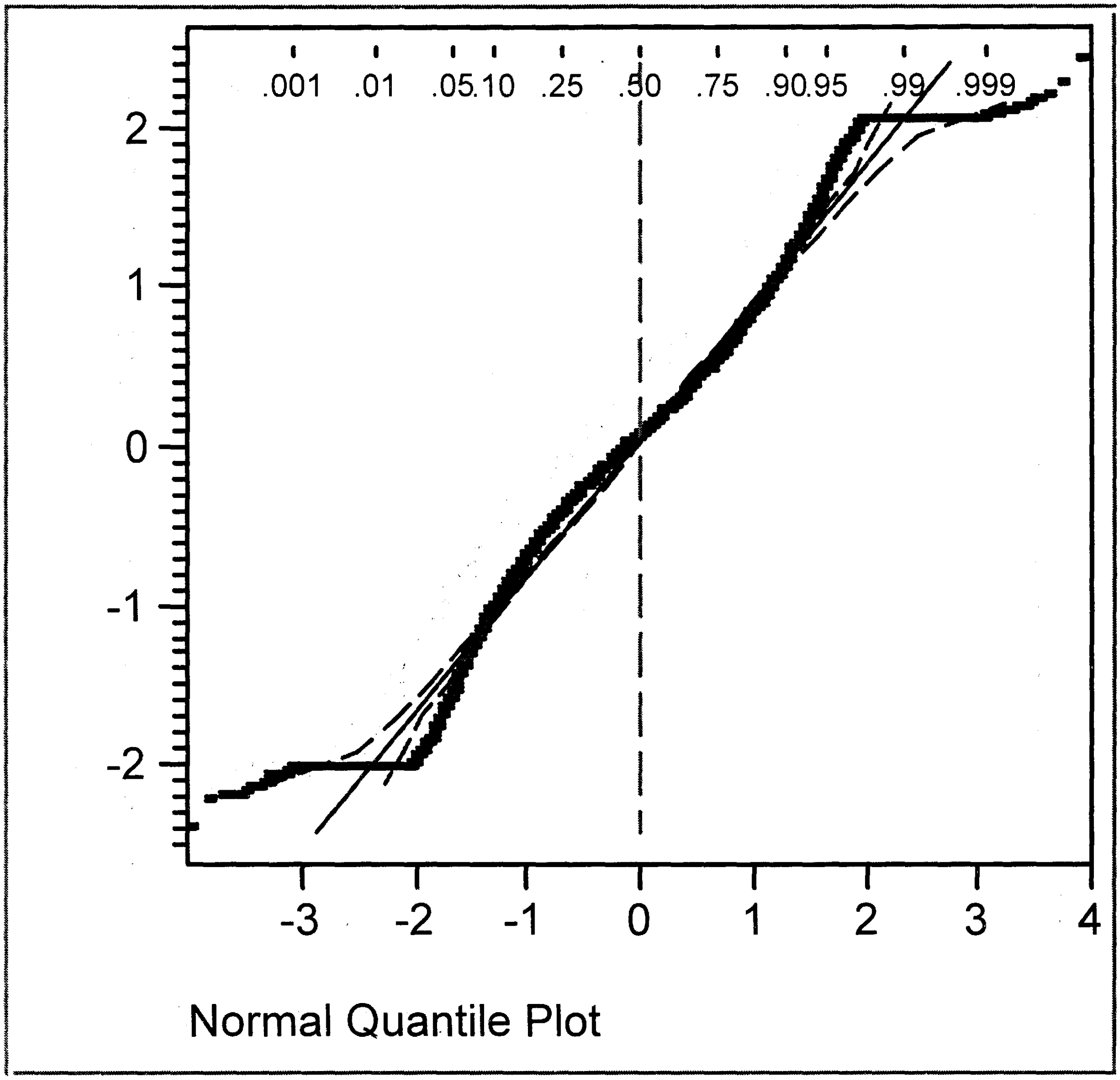

Figure 32. Normal probability plot for daily price changes on reduced all contracts

The values of the skewness and the kurtosis and the significance level of their test statistic for February, April, June, July, August, October, December and All-Contracts contracts are given in Table 9, Table 10, Table 11, Table 12, Table 13, Table 14, Table 15 and Table 16 respectively. The skewness is statistically not different from zero for April, June, July, October and December contracts but statistically different from zero for February, August and All-Contracts contracts. The kurtosis is statistically different from three for all those contracts except for the December contract.

The normal probability plots for February, April, June, July, August, October, December and All-Contracts contracts are given in Figure 18, Figure 20, Figure 22, Figure 24, Figure 26, Figure 28, Figure 30 and Figure 32 respectively. All those normal probability plots suggest that the distribution of daily price changes in the live hog futures is not normal.

The mean daily price changes and the significance level of their sample statistic for February, April, June, July, August, October, December and All-Contracts contracts are given in Table 9, Table 10, Table 11, Table 12, Table 13, Table 14, Table 15 and Table 16 respectively. The mean is statistically not different from zero for February, April, October and December contracts but statistically different from zero for June, July, August, and All-Contracts contracts.

The sample variance of daily price changes for February, April, June, July, August, October, December and All-Contracts contracts are 0.704792, 0.637380, 0.716947, 0.715327, 0.769993, 0.757642, 0.816322 and 0.730699 respectively. The variability in the daily price changes seems to be little bit inconsistent but, perhaps, can be ignored.

Thus there appears to be significant evidence that the daily price changes in the live hog futures are not normally distributed and don't have zero mean either.

2.3 Alternate Data and Analysis

Since there is a limit set by the exchange on the change in daily prices, the daily price change may not be a good indicator of the change in price of the futures. For that reason, the six-month change in price for all the contracts was calculated. This was computed as the difference between the price of the futures on the date of expiration and the price of the futures on a date six months prior to the date of expiration. This data set had 193 observations.

Summary statistics for this data set are given in Table 17. The skewness is not statistically different from zero, the kurtosis is not statistically different from three but the mean is statistically different from zero.

The histogram with normal curve of this data is given in Figure 33. The histogram shows that the distribution has more mass in the center than it should have under normality assumption.

The normal probability plot of this data is given in Figure 34. It shows that the distribution is normal.

The sample variance of the six-month change in price is 103.593169.

Thus the distribution of six-month change in price of live hog futures seems to be normal with non-zero mean and large variance.

The findings in this section suggest that the distribution of futures prices may not be lognormal. Thus it should be of great academic interest to consider both lognormal and some non-lognormal distributions of the futures price and derive, examine and compare the prices of the options on futures with these distributions.

Table 17. Statistics of the six monthly price changes on all contracts

Variable	N	Mean	Median	TrMean	StDev
6MosPriceChange	193	3.780	2.633	3.774	10.178

Variable	SE Mean	Minimum	Maximum	Q1	Q3
6MosPriceChange	0.733	-21.425	33.305	-2.585	9.936

Observations = 193

Sample Mean	=	3.7802412435	Variance	=	103.593169
Standard Error	=	10.1780729344	SE of Sample Mean	=	0.732634
t-Statistic	=	5.15980	Signif Level (Mean=0)	=	0.00000061
Skewness	=	0.11881	Signif Level (Sk=0)	=	0.50375074
Kurtosis	=	-0.00376	Signif Level (Ku=0)	=	0.99163765

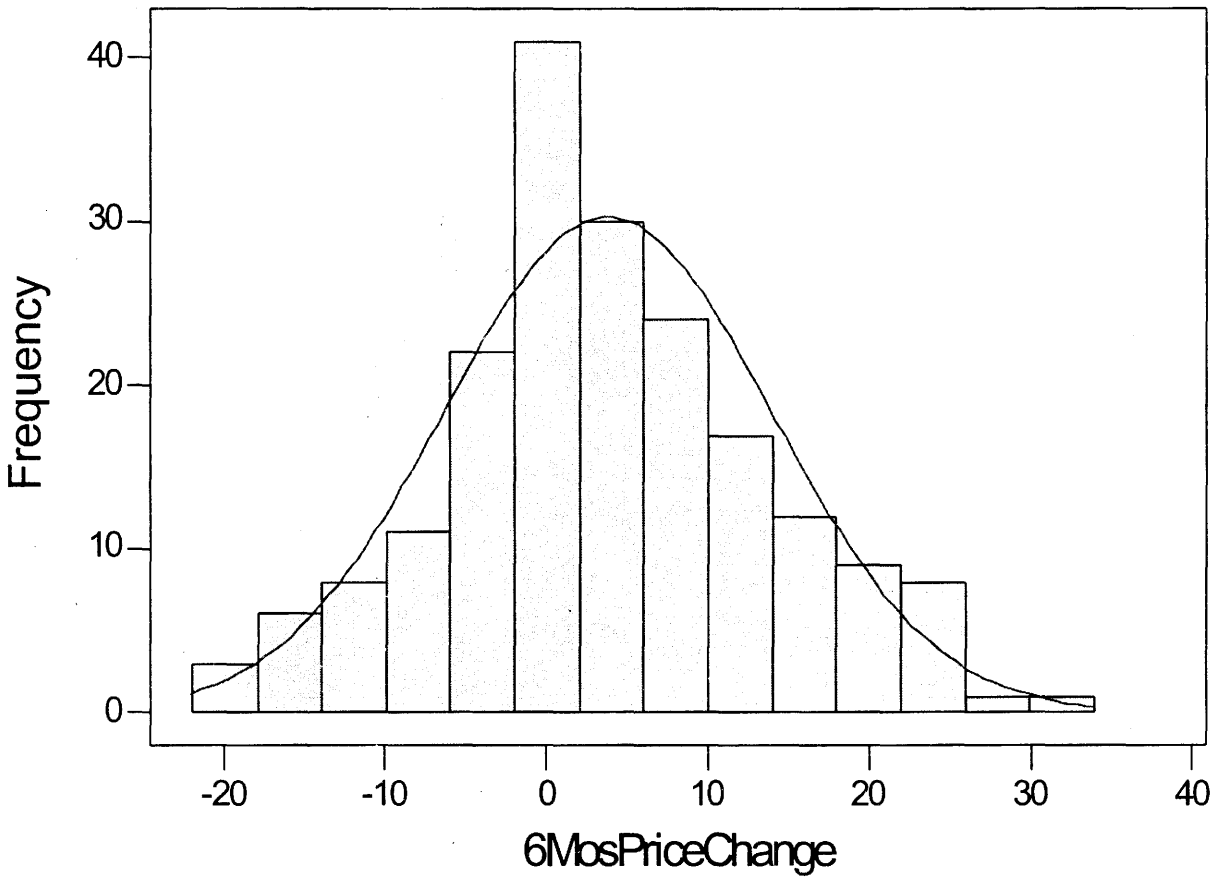

Figure 33. Histogram with normal curve for six monthly price changes on all contracts

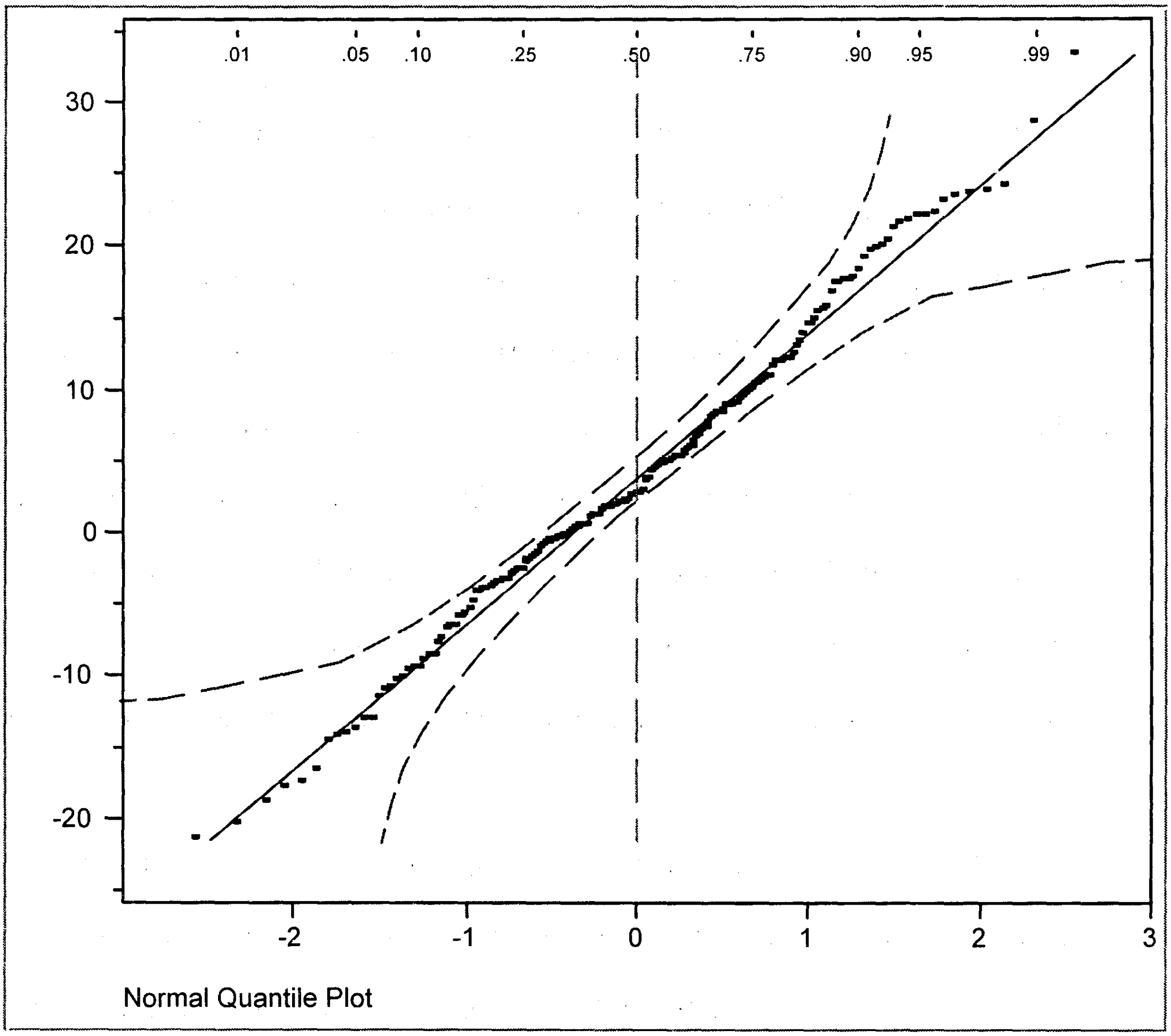

Figure 34. Normal probability plot for six monthly price changes on all contracts

3. METHODOLOGY

Two types of approaches, the Theoretical Approach and the Simulation Approach, of pricing European type call options are discussed here. The Theoretical Approach gives, using the theoretical understanding of the mechanism of option pricing, a single algebraic formula for the value of the option and the numerical value of the option can be obtained by simply plugging in the value of the unknown quantities in that algebraic formula. The Simulation Approach simulates the futures prices with desired probability distribution, compares those prices with the strike price and computes option premium for each of those prices. The option value is calculated as the average of all those call premiums. As can be understood, the accuracy of the option value will increase with the increase in the size of the simulated futures prices.

To formulate the Theoretical Approach requires the total understanding of the mechanism of option pricing as well as the mathematical concepts involved whereas to formulate the Simulation Approach needs the understanding of the mechanism of option pricing only.

The distributions of futures prices that were considered here are Beta, Gamma, Lognormal and Uniform. The price of the futures contract at time zero was assumed to be "**pbarc**" and the volatility of the futures price was assumed to be "**cvpc**". Thus the terms "**pbarc**" and "**cvpc**" were used to denote the mean and the variance of the distributions respectively. In order to keep things simple, option calculations were done for and at time t and t was assumed to be one.

3.1 Theoretical Approach

The value of a European type call option is nothing but the conditional mean of the futures price given that the call option is in-the-money or the futures price is greater than the strike price.

3.1.1 Lognormal distribution

The lognormal distribution has two parameters μ and σ^2, which have to be estimated by using two known quantities "**pbarc**", and "**cvpc**". Here "**pbarc**" and "**cvpc**" are considered to be the mean and variance of the futures price. μ and σ^2 can be calculated by equating the mean and variance of a lognormal distribution with "**pbarc**" and "**cvpc**" respectively as shown below.

Probability Density Function of Lognormal distribution is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \frac{\exp\left[-(\log x - \mu)^2 / 2\sigma^2\right]}{x} \quad \text{for } 0 \le x \le \infty$$

$$\text{Mean} = E(X) = \text{pbarc} = e^{\mu + \frac{\sigma^2}{2}}$$

Taking log on both sides, we get,

$$\log EX = \log(\text{pbarc}) = \mu + \frac{\sigma^2}{2} \Rightarrow \mu = \log(\text{pbarc}) - \frac{\sigma^2}{2}$$

$$\text{Variance} = V(X) = \text{cvpc} = e^{2(\mu+\sigma^2)} - e^{2\mu+\sigma^2} = e^{2\mu+\sigma^2}\left(e^{\sigma^2} - 1\right) = (EX)^2\left(e^{\sigma^2} - 1\right)$$

$$\Rightarrow \text{cvpc} = (\text{pbarc})^2 * \left(e^{\sigma^2} - 1\right)$$

$$\Rightarrow\left(\frac{cvpc}{pbarc^2}+1\right)=e^{\sigma^2}\Rightarrow\sigma^2=\log\left(\frac{cvpc}{pbarc^2}+1\right)$$

Once the value of μ and σ^2 have been calculated and known, the price of the option can be calculated as shown below.

$$\text{Option Price}=\int_K^\infty(s-K)f(s)ds=\int_K^\infty(s-K)\frac{1}{\sigma\sqrt{2\pi}}\frac{\exp[-(\log s-\mu)^2/2\sigma^2]}{s}ds$$

$$=\frac{1}{\sigma\sqrt{2\pi}}\left[\int_K^\infty\exp[-(\log s-\mu)^2/2\sigma^2]ds-K\int_K^\infty\frac{\exp[-(\log s-\mu)^2/2\sigma^2]}{s}ds\right]$$

Let, $\frac{\log s-\mu}{\sigma}=x \quad \Rightarrow \frac{1}{s}ds=\sigma dx$

Also, $\log s=\mu+x\sigma\Rightarrow s=e^{\mu+x\sigma}$

$s=K\Rightarrow x=\frac{\log K-\mu}{\sigma}$ and $s=\infty\Rightarrow x=\infty$

$$\text{Hence we get, Option Price}=\frac{1}{\sqrt{2\pi}}\left[\int_{\frac{\log K-\mu}{\sigma}}^\infty e^{\frac{-x^2}{2}}e^{\mu+\sigma x}dx-K\int_{\frac{\log K-\mu}{\sigma}}^\infty e^{\frac{-x^2}{2}}dx\right]$$

$$=\frac{1}{\sqrt{2\pi}}\left[\int_{\frac{\log K-\mu}{\sigma}}^\infty e^{\frac{-x^2}{2}}e^{\mu+\frac{\sigma^2}{2}}e^{\sigma x-\frac{\sigma^2}{2}}dx\right]-K\left[\frac{1}{\sqrt{2\pi}}\int_{\frac{\log K-\mu}{\sigma}}^\infty e^{\frac{-x^2}{2}}dx\right]$$

$$=e^{\mu+\frac{\sigma^2}{2}}\frac{1}{\sqrt{2\pi}}\int_{\frac{\log K-\mu}{\sigma}}^\infty e^{\frac{-(x-\sigma)^2}{2}}dx-K*\text{Prob}\left(z>\frac{\log K-\mu}{\sigma}\right)$$

Let, $x-\sigma=z\Rightarrow dx=dz$

$$\text{Option Price} = e^{\mu+\frac{\sigma^2}{2}} \frac{1}{\sqrt{2\pi}} \int_{\frac{\log K-\mu}{\sigma}-\sigma}^{\infty} e^{\frac{-z^2}{2}} dz - K * \text{Prob}\left(z > \frac{\log K - \mu}{\sigma}\right)$$

$$= e^{\mu+\frac{\sigma^2}{2}} * \text{Prob}\left(z > \left(\frac{\log K - \mu}{\sigma} - \sigma\right)\right) - K * \text{Prob}\left(z > \frac{\log K - \mu}{\sigma}\right)$$

$$= \text{pbarc} * \text{Prob}\left(z > \left(\frac{\log K - \mu}{\sigma} - \sigma\right)\right) - K * \text{Prob}\left(z > \frac{\log K - \mu}{\sigma}\right)$$

$$= \text{pbarc} * \text{Prob}\left(z < \left(-\frac{\log K - \mu}{\sigma} + \sigma\right)\right) - K * \text{Prob}\left(z < -\frac{\log K - \mu}{\sigma}\right)$$

We find that, $-\frac{\log K - \mu}{\sigma} = -\frac{\log K - \log \text{pbarc} + \sigma^2/2}{\sigma} = \frac{1}{\sigma}\log(\text{pbarc}/K) - \frac{1}{2}\sigma$

Therefore, $\text{Option Price} = \text{pbarc} * \varphi\left(\frac{1}{\sigma}\log(\text{pbarc}/K) + \frac{1}{2}\sigma\right) - K * \varphi\left(\frac{1}{\sigma}\log(\text{pbarc}/K) - \frac{1}{2}\sigma\right)$

Where φ stands for cumulative standard normal probability function. This can be found by using the function NORMSDIST in EXCEL. The function NORMSDIST in EXCEL has one argument, z. For the expression $\varphi\left(\frac{1}{\sigma}\log(\text{pbarc}/K) + \frac{1}{2}\sigma\right)$, $z = \left(\frac{1}{\sigma}\log(\text{pbarc}/K) + \frac{1}{2}\sigma\right)$.

3.1.2 Beta distribution

The beta distribution has four parameters α, β, **a** and **b** which have to be estimated by using only two known quantities "**pbarc**" and "**cvpc**". α and β can be calculated after knowing the value of **a** and **b**. Parameters **a** and **b** are the lower and upper limit of the futures price and can be calculated using two quantities **delmaxc** and **delminc** which are defined as shown below.

delmaxc = number of standard deviation away from the mean on the upper side.

delminc = number of standard deviation away from the mean on the lower side.

The value of **delmaxc** is the number of standard deviation away from the mean on the upper side. The value of **delminc** is the number of standard deviation away from the mean on the lower side. The value of mean and standard deviation here are "pbarc" and $\sqrt{\text{cvpc}}$ respectively. If the difference between "**pbarc**" and the quantity (**delminc** * $\sqrt{\text{cvpc}}$) is less than zero then **a** is set to zero otherwise **a** is calculated as the quantity

(**pbarc** - **delminc** * $\sqrt{\text{cvpc}}$). **b** is calculated as the quantity (**pbarc** + **delmaxc** * $\sqrt{\text{cvpc}}$).

Once the value of **a** and **b** are known, α and β can be calculated by equating the mean and variance of a Beta distribution with "**pbarc**" and "**cvpc**" respectively as shown below.

Probability Density Function of Beta distribution is:

$$f(x) = \frac{1}{B(\alpha,\beta)} \frac{(x-a)^{\alpha-1}(b-x)^{\beta-1}}{(b-a)^{\alpha+\beta-1}} \quad \text{for } a \le x \le b$$

$$\text{Mean} = E(X) = (b-a)\frac{\alpha}{\alpha+\beta} + a \quad \Rightarrow \text{pbarc} = (b-a)\frac{\alpha}{\alpha+\beta} + a$$

$$\Rightarrow \text{pbarc} - a = (b-a)\frac{\alpha}{\alpha+\beta} \quad \Rightarrow \frac{\text{pbarc}-a}{b-a} = \frac{\alpha}{\alpha+\beta}$$

$$\Rightarrow \frac{b-a}{\text{pbarc}-a} = \frac{\alpha+\beta}{\alpha} \quad \Rightarrow \frac{b-\text{pbarc}}{\text{pbarc}-a} = \frac{\beta}{\alpha}$$

$$\text{Variance} = V(X) = \frac{(b-a)^2\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} \quad \Rightarrow \text{cvpc} = \frac{(b-a)^2\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

Also, $$\frac{(pbarc-a)^2}{cvpc} = \frac{(b-a)^2\alpha^2}{(\alpha+\beta)^2} * \frac{(\alpha+\beta)^2(\alpha+\beta+1)}{(b-a)^2\alpha\beta} = \frac{\alpha}{\beta}(\alpha+\beta+1)$$

$$\Rightarrow \frac{pbarc-a}{b-pbarc}\left(\frac{b-a}{pbarc-a}*\alpha+1\right) = \frac{(pbarc-a)^2}{cvpc}$$

$$\Rightarrow (b-a)\alpha+(pbarc-a) = (b-pbarc)*\frac{(pbarc-a)^2}{cvpc}$$

$$\Rightarrow \alpha = \frac{pbarc-a}{b-a}\left[\frac{(b-pbarc)(pbarc-a)}{cvpc}-1\right] \quad \text{and}$$

$$\beta = \frac{b-pbarc}{b-a}\left[\frac{(b-pbarc)(pbarc-a)}{cvpc}-1\right]$$

Once the value of α, β, **a** and **b** have been calculated and known, the price of the option can be calculated as shown below.

$$\text{Option Price} = \int_K^{\infty}(s-K)f(s)ds = \int_K^{b}(s-K)f(s)ds$$

$$= \int_K^{b}(s-K)\frac{1}{B(\alpha,\beta)}\frac{(s-a)^{\alpha-1}(b-s)^{\beta-1}}{(b-a)^{\alpha+\beta-1}}ds \qquad \text{for } a \le s \le b$$

Let's assume, $y = \dfrac{s-a}{b-a} \quad \Rightarrow s = a+(b-a)y \quad \Rightarrow ds = (b-a)dy$

$$s = K \quad \Rightarrow y = \frac{K-a}{b-a} \quad \Rightarrow (1-y) = \frac{K-a}{b-a}$$

$$s = b \quad \Rightarrow y = 1$$

We find, $$\text{Option Price} = \int_{\frac{K-a}{b-a}}^{1}\left[(b-a)y-(K-a)\right]\frac{1}{B(\alpha,\beta)}y^{\alpha-1}(1-y)^{\beta-1}dy$$

$$= \frac{1}{B(\alpha,\beta)}\left[(b-a)\int_{\frac{K-a}{b-a}}^{1} y^{\alpha}(1-y)^{\beta-1}dy - (K-a)\int_{\frac{K-a}{b-a}}^{1} y^{\alpha-1}(1-y)^{\beta-1}dy\right]$$

$$= \frac{B(\alpha+1,\beta)}{B(\alpha,\beta)} * (b-a) * \frac{1}{B(\alpha+1,\beta)}\int_{\frac{K-a}{b-a}}^{1} y^{\alpha}(1-y)^{\beta-1}dy$$

$$-(K-a) * \frac{1}{B(\alpha,\beta)}\int_{\frac{K-a}{b-a}}^{1} y^{\alpha-1}(1-y)^{\beta-1}dy$$

Now, $\frac{B(\alpha+1,\beta)}{B(\alpha,\beta)} = \frac{\Gamma(\alpha+1)\Gamma(\beta)}{\Gamma(\alpha+\beta+1)} * \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} = \frac{\alpha * \Gamma(\alpha) * \Gamma(\beta) * \Gamma(\alpha+\beta)}{(\alpha+\beta) * \Gamma(\alpha+\beta) * \Gamma(\alpha) * \Gamma(\beta)} = \frac{\alpha}{\alpha+\beta}$

Hence we find, Option Price =

$$(b-a) * \frac{\alpha}{\alpha+\beta} * \frac{1}{B(\alpha+1,\beta)}\int_{\frac{K-a}{b-a}}^{1} y^{\alpha+1-1}(1-y)^{\beta-1}dy - (K-a) * \frac{1}{B(\alpha,\beta)}\int_{\frac{K-a}{b-a}}^{1} y^{\alpha-1}(1-y)^{\beta-1}dy$$

$$= (b-a) * \frac{\alpha}{\alpha+\beta} * \mathrm{Prob}\left(u > \frac{K-a}{b-a}\right) - (K-a) * \mathrm{Prob}\left(v > \frac{K-a}{b-a}\right)$$

$$= (b-a) * \frac{\alpha}{\alpha+\beta} * \left[1 - \mathrm{Prob}\left(u \le \frac{K-a}{b-a}\right)\right] - (K-a) * \left[1 - \mathrm{Prob}\left(v \le \frac{K-a}{b-a}\right)\right]$$

Where, $u \approx \mathrm{Beta}(\alpha+1,\beta)$ and $v \approx \mathrm{Beta}(\alpha,\beta)$

$$\text{Therefore, Option Price} = (b-a) * \frac{\alpha}{\alpha+\beta} * \left[1 - I_{\frac{K-a}{b-a}}(\alpha+1,\beta)\right] - (K-a) * \left[1 - I_{\frac{K-a}{b-a}}(\alpha,\beta)\right]$$

$$= (\text{pbarc} - a) * \left[1 - I_{\frac{K-a}{b-a}}(\alpha+1,\beta)\right] - (K-a) * \left[1 - I_{\frac{K-a}{b-a}}(\alpha,\beta)\right]$$

where, I stands for Incomplete Beta Function Ratio. This can be found by using the function BETADIST in EXCEL. The function BETADIST in EXCEL has five arguments: x, alpha, beta, A and B. For the incomplete beta function $I_{\frac{K-a}{b-a}}(\alpha+1,\beta)$,

$x = \dfrac{K-a}{b-a}$, alpha = $\alpha + 1$, beta = β, A = 0, and B = 1.

3.1.3 Uniform distribution

The uniform distribution has two parameters **d** and **c** which can be calculated by equating the mean and variance of a Uniform distribution with "**pbarc**" and "**cvpc**" respectively as shown below.

Probability Density Function of Uniform distribution is:

$$f(x) = \frac{1}{d-c} \qquad \text{for } c \le x \le d$$

$$\text{Mean} = E(X) = \frac{d+c}{2} \quad \Rightarrow \text{pbarc} = \frac{d+c}{2} \quad \Rightarrow d + c = 2\text{pbarc}$$

$$\text{Variance} = V(X) = \frac{(d-c)^2}{12} \quad \Rightarrow \text{cvpc} = \frac{(d-c)^2}{12} \quad \Rightarrow d - c = 2\sqrt{3\text{cvpc}}$$

$$\Rightarrow d = \frac{1}{2}\left(2\text{pbarc} + 2\sqrt{3\text{cvpc}}\right) = \text{pbarc} + \sqrt{3\text{cvpc}}$$

$$\Rightarrow c = \frac{1}{2}\left(2\text{pbarc} - 2\sqrt{3\text{cvpc}}\right) = \text{pbarc} - \sqrt{3\text{cvpc}}$$

Once the value of d and c have been calculated and known, the price of the option can be calculated as shown below.

$$\text{Option Price} = \int_K^{\infty}(s-K)f(s)ds = \int_K^{d}(s-K)f(s)ds = \frac{1}{d-c}\int_K^{d}(s-K)ds$$

$$= \frac{1}{2(d-c)}\left[(s-K)^2\right]_K^d = \frac{(d-K)^2}{2(d-c)} = \frac{1}{4\sqrt{3V}}\left(pbarc + \sqrt{3cvpc} - K\right)^2 \quad \text{or}$$

$$= pbarc * Prob(u > K^2) - K * Prob(v > K)$$

where, $u \approx \text{Uniform}(c^2, d^2)$ and $v \approx \text{Uniform}(c, d)$

3.1.4 Gamma distribution

Gamma distribution has two parameters α and β which have to be estimated by using two known quantities "**pbarc**" and "**cvpc**". The value of α and β can be calculated by equating the mean and variance of a Gamma distribution with "**pbarc**" and "**cvpc**" respectively as shown below.

Probability Density Function of Gamma distribution is:

$$f(x) = \frac{1}{\Gamma(\alpha) * \beta^{\alpha}} x^{\alpha-1} e^{-x/\beta} \qquad \text{for } 0 \le x \le \infty$$

$$\text{Mean} = E(X) = \alpha\beta \quad \Rightarrow pbarc = \alpha\beta$$

$$\text{Variance} = V(X) = \alpha\beta^2 \quad \Rightarrow cvpc = \alpha\beta^2$$

$$\Rightarrow \frac{cvpc}{pbarc} = \frac{\alpha\beta^2}{\alpha\beta} = \beta \Rightarrow \beta = \frac{cvpc}{pbarc}$$

$$\text{Also, } pbarc = \alpha\beta \Rightarrow \alpha = \frac{pbarc}{\beta}$$

$$\Rightarrow \alpha = \frac{pbarc^2}{cvpc}$$

Once the value of α and β have been calculated and known, the price of the option can be calculated as shown below.

$$\text{Option Price} = \int_K^\infty (s-K)f(s)ds = \int_K^\infty (s-K)\frac{1}{\Gamma(\alpha)*\beta^\alpha}s^{\alpha-1}e^{-s/\beta}ds$$

$$= \frac{1}{\Gamma(\alpha)*\beta^\alpha}\int_K^\infty s^\alpha e^{-s/\beta}ds - K*\frac{1}{\Gamma(\alpha)*\beta^\alpha}\int_K^\infty s^{\alpha-1}e^{-s/\beta}ds$$

$$= \frac{\beta*\Gamma(\alpha+1)}{\Gamma(\alpha)}*\frac{1}{\Gamma(\alpha+1)*\beta^{\alpha+1}}\int_K^\infty s^{\alpha+1-1}e^{-s/\beta}ds - K*\frac{1}{\Gamma(\alpha)*\beta^\alpha}\int_K^\infty s^{\alpha-1}e^{-s/\beta}ds$$

$$= \alpha\beta*\frac{1}{\Gamma(\alpha+1)*\beta^{\alpha+1}}\int_K^\infty s^{\alpha+1-1}e^{-s/\beta}ds - K*\frac{1}{\Gamma(\alpha)*\beta^\alpha}\int_K^\infty s^{\alpha-1}e^{-s/\beta}ds$$

$$= \text{pbarc}*\text{Prob}(u > K) - K*\text{Prob}(v > K)$$

$$= \text{pbarc}*[1-\text{Prob}(u \le K)] - K*[1-\text{Prob}(v \le K)]$$

where, $u \approx \text{Gamma}(\alpha+1,\beta)$ and $v \approx \text{Gamma}(\alpha,\beta)$

$$\Rightarrow \text{Option Price} = \text{pbarc}*[1-I_K(\alpha+1,\beta)] - K*[1-I_K(\alpha,\beta)]$$

where, I stands for Incomplete Gamma Function Ratio. This can be found by using the function GAMMADIST in EXCEL. The function GAMMADIST in EXCEL has four arguments: x, alpha, beta and cumulative. For the incomplete gamma function $I_K(\alpha+1,\beta)$, x = K, alpha = α + 1, beta = β.

cumulative is a logical value. If the value is given as TRUE then it returns the cumulative probability distribution function, if the value is given as FALSE then it returns the probability mass function. In this case, the value would be TRUE.

3.2 Generalization of the Theoretical Findings

As mentioned before, in order to keep things simple, it was assumed that all option calculations were at time t and t was equal to one. It is not very difficult to calculate the option price at time 0 for any value of t.

For lognormal distribution the option price that were calculated at time t for t=1 was as below,

$$\text{Option Price} = \text{pbarc} * \varphi\left(\frac{1}{\sigma}\log(\text{pbarc}/K) + \frac{1}{2}\sigma\right) - K * \varphi\left(\frac{1}{\sigma}\log(\text{pbarc}/K) - \frac{1}{2}\sigma\right)$$

For any t other than unity, the standard deviation would be equal to $\sigma\sqrt{t}$ and after replacing σ with $\sigma\sqrt{t}$ the above formula would look like

$$\text{Option Price} = \text{pbarc} * \varphi\left(\frac{1}{\sigma\sqrt{t}}\log(\text{pbarc}/K) + \frac{1}{2}\sigma\sqrt{t}\right) - K * \varphi\left(\frac{1}{\sigma\sqrt{t}}\log(\text{pbarc}/K) - \frac{1}{2}\sigma\sqrt{t}\right)$$

This price is for at time t. This price has to be discounted at the risk-free rate in order to calculate the option price at time zero. The option price at time zero would be as below.

$$\text{Option Price} =$$

$$\exp(-rt) * \left[\text{pbarc} * \phi\left(\frac{1}{\sigma\sqrt{t}}\log(\text{pbarc}/K) + \frac{1}{2}\sigma\sqrt{t}\right) - K * \phi\left(\frac{1}{\sigma\sqrt{t}}\log(\text{pbarc}/K) - \frac{1}{2}\sigma\sqrt{t}\right)\right]$$

The algebraic expression above is nothing but Black's formula for calculating the price of options on futures and it also assumes the lognormal distribution of futures price. It is possible, in a similar fashion, to calculate option prices for any probability distribution of the futures for time t at time zero.

3.3 Simulation Approach

5000 normal random numbers were generated using EXCEL. The futures prices with Lognormal, Beta, Gamma and Uniform distributions were generated using those random numbers. The price of the futures contract for the current period was assumed to be "**pbarc**" and the volatility of the futures price was assumed to be "**cvpc**". Thus prices with different distributions but same mean and variance were generated. It was assumed that these constant mean and variance of the price were known. The terms "**pbarc**" and "**cvpc**" were used to denote the mean and the variance of the distributions respectively. In order to keep things simple, option calculations were done for and at time t and t was assumed to be one.

3.3.1 Generation of futures prices

Futures prices with Beta, Gamma, Lognormal and Uniform probability distribution were generated by using EXCEL as described in the following sections.

3.3.1.1 Lognormal distribution

The two unknown parameters of the lognormal distribution, μ and σ^2 were calculated equating the mean and variance of the lognormal distribution with "**pbarc**" and "**cvpc**" respectively as shown below.

$$\text{Probability Density Function} = f(x) = \frac{1}{\sigma\sqrt{2\pi}} \frac{\exp\left[-(\log x - \mu)^2 / 2\sigma^2\right]}{x} \quad \text{for } 0 \le x \le \infty$$

$$\text{Mean} = E(X) = \text{pbarc} = e^{\mu + \frac{\sigma^2}{2}}$$

$$\text{Variance} = V(X) = \text{cvpc} = e^{2(\mu+\sigma^2)} - e^{2\mu+\sigma^2} = e^{2\mu+\sigma^2}\left(e^{\sigma^2} - 1\right) = (EX)^2\left(e^{\sigma^2} - 1\right)$$

$$\log EX = \log(\text{pbarc}) = \mu + \frac{\sigma^2}{2} \Rightarrow \mu = \log(\text{pbarc}) - \frac{\sigma^2}{2}$$

$$\left(\frac{\text{cvpc}}{\text{pbarc}^2} + 1\right) = e^{\sigma^2} \Rightarrow \sigma^2 = \log\left(\frac{\text{cvpc}}{\text{pbarc}^2} + 1\right)$$

The following EXCEL codes were used to calculate μ, σ^2 and to generate the prices.

sigmac = LN((cvpc / (pbarc ^ 2)) + 1)

muc= LN(pbarc) – sigmac / 2

Price=LOGINV(NORMSDIST(corresponding random number), muc, SQRT(sigmac))

The terms "**muc**" and "**sigmac**" were used to denote μ and σ^2 respectively.

3.3.1.2 Beta distribution

Beta distribution has four parameters α, β, **a** and **b** which were estimated by using only two known quantities "**pbarc**" and "**cvpc**". Considering **a** and **b** to be known, α and β were calculated as shown below.

$$\text{Probability Density Function} = f(x) = \frac{1}{B(\alpha,\beta)} \frac{(x-a)^{\alpha-1}(b-x)^{\beta-1}}{(b-a)^{\alpha+\beta-1}} \quad \text{for } a \le x \le b$$

$$\text{Mean} = E(X) = (b-a)\frac{\alpha}{\alpha+\beta} + a \Rightarrow \text{pbarc} = (b-a)\frac{\alpha}{\alpha+\beta} + a$$

$$\Rightarrow \text{pbarc} - a = (b-a)\frac{\alpha}{\alpha+\beta} \Rightarrow \frac{\text{pbarc}-a}{b-a} = \frac{\alpha}{\alpha+\beta}$$

$$\Rightarrow \frac{b-a}{\text{pbarc}-a} = \frac{\alpha+\beta}{\alpha} \Rightarrow \frac{b-\text{pbarc}}{\text{pbarc}-a} = \frac{\beta}{\alpha}$$

$$\text{Variance} = V(X) = \frac{(b-a)^2\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} \quad \Rightarrow \text{cvpc} = \frac{(b-a)^2\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

$$\text{Also, } \frac{(\text{pbarc}-a)^2}{\text{cvpc}} = \frac{(b-a)^2\alpha^2}{(\alpha+\beta)^2} * \frac{(\alpha+\beta)^2(\alpha+\beta+1)}{(b-a)^2\alpha\beta} = \frac{\alpha}{\beta}(\alpha+\beta+1)$$

$$\Rightarrow \frac{\text{pbarc}-a}{b-\text{pbarc}}\left(\frac{b-a}{\text{pbarc}-a} * \alpha + 1\right) = \frac{(\text{pbarc}-a)^2}{\text{cvpc}}$$

$$\Rightarrow (b-a)\alpha + (\text{pbarc}-a) = (b-\text{pbarc}) * \frac{(\text{pbarc}-a)^2}{\text{cvpc}}$$

$$\Rightarrow \alpha = \frac{\text{pbarc}-a}{b-a}\left[\frac{(b-\text{pbarc})(\text{pbarc}-a)}{\text{cvpc}} - 1\right] \quad \text{and}$$

$$\beta = \frac{b-\text{pbarc}}{b-a}\left[\frac{(b-\text{pbarc})(\text{pbarc}-a)}{\text{cvpc}} - 1\right]$$

Parameters **a** and **b** were assumed to be known and were calculated as below.

delmaxc = number of standard deviation away from the mean on the upper side.

delminc = number of standard deviation away from the mean on the lower side.

a = IF((pbarc – delminc * SQRT(cvpc)) < 0, 0, (pbarc - delminc * SQRT(cvpc)))

b = pbarc + delmaxc * SQRT(cvpc)

The following EXCEL codes were used to calculate α, β and to generate the prices.

alpha = ((pbarc - a) / (b - a)) * (((b - pbarc) * (pbarc - a) / cvpc) - 1)

beta = alpha * (b - pbarc) / (pbarc - a)

Price = BETAINV(NORMSDIST(corresponding random number), alpha, beta, a, b)

The terms "**alpha**" and "**beta**" were used to denote α and β respectively.

3.3.1.3 Uniform distribution

Unknown parameters **d** and **c** were estimated as shown below.

$$\text{Probability Density Function} = f(x) = \frac{1}{d-c} \qquad \text{for } c \le x \le d$$

$$\text{Mean} = E(X) = \frac{d+c}{2} \quad \Rightarrow \text{pbarc} = \frac{d+c}{2} \quad \Rightarrow d + c = 2\text{pbarc}$$

$$\text{Variance} = V(X) = \frac{(d-c)^2}{12} \quad \Rightarrow \text{cvpc} = \frac{(d-c)^2}{12} \quad \Rightarrow d - c = 2\sqrt{3\text{cvpc}}$$

$$\Rightarrow d = \frac{1}{2}\left(2\text{pbarc} + 2\sqrt{3\text{cvpc}}\right) = \text{pbarc} + \sqrt{3\text{cvpc}}$$

$$\Rightarrow c = \frac{1}{2}\left(2\text{pbarc} - 2\sqrt{3\text{cvpc}}\right) = \text{pbarc} - \sqrt{3\text{cvpc}}$$

The following EXCEL codes were used to calculate c, d and to generate the prices.

upper = pbarc + (3 * cvpc) ^ 0.5

lower = IF((pbarc - (3 * cvpc) ^ 0.5) < 0, 0, (pbarc - (3 * cvpc) ^ 0.5))

Price = (NORMSDIST(corresponding random number)) * (upper - lower) + lower

The terms "**upper**" and "**lower**" were used to denote **d** and c respectively.

3.3.1.4 Gamma distribution

Unknown parameters α and β were estimated as shown below.

$$\text{Probability Density Function} = f(x) = \frac{1}{\Gamma(\alpha) * \beta^{\alpha}} x^{\alpha-1} e^{-x/\beta} \qquad \text{for } 0 \le x \le \infty$$

$$\text{Mean} = E(X) = \alpha\beta \quad \Rightarrow \text{pbarc} = \alpha\beta$$

$$\text{Variance} = V(X) = \alpha\beta^2 \quad \Rightarrow \text{cvpc} = \alpha\beta^2$$

$$\Rightarrow \frac{\text{cvpc}}{\text{pbarc}} = \frac{\alpha\beta^2}{\alpha\beta} = \beta \Rightarrow \beta = \frac{\text{cvpc}}{\text{pbarc}}$$

Also, $\text{pbarc} = \alpha\beta \Rightarrow \alpha = \dfrac{\text{pbarc}}{\beta}$

$$\Rightarrow \alpha = \frac{\text{pbarc}^2}{\text{cvpc}}$$

The following EXCEL codes were used to calculate α, β and to generate the prices.

galpha = pbarc / gbeta

gbeta = cvpc / pbarc

Price = GAMMAINV(NORMSDIST(corresponding random number), galpha, gbeta)

The terms "**galpha**" and "**gbeta**" were used to denote α and β respectively.

3.3.2 Calculation of option prices from simulated futures prices

The type of option that was considered here is European type call option. The simulation of the prices has been discussed in the preceding sections. Each of those simulated prices was compared to the strike price. If the strike price was less than the simulated price then the call premium was set to zero. If the simulated price was larger than the strike price then the call premium was calculated to be the difference between the simulated price and the strike price. Similarly, the call premium was calculated for all those 5000 simulated prices. The call option price on such a futures contract is nothing but the average of those 5000 call premiums. Thus the call option prices were calculated for Lognormal, Beta, Uniform and Gamma distributions.

3.3.3 Discussion on the simulation approach

The simulation technique seems to be very easy to understand and use. Due to it's open architecture it can be used for any probability distribution. Black-Scholes formulated a theoretical model for the lognormal distribution which required the understanding of complicated return structure of the underlying asset and complex mathematical concepts like diffusion process and Brownian motion. The simulation technique requires the understanding of the price process only. The prices can be generated by using EXCEL or any other software if the probability distribution of the price and the parameters of the distribution are known. The pricing of the option from the prices is then just routine computation as discussed before.

3.4 Discussion on the Simulation and Theoretical Approach

It has been shown how to calculate the price of the European type call option by using Simulation Approach and the Theoretical Approach when the underlying asset has Lognormal, Beta, Uniform or Gamma distribution. It has also been shown how theoretically Black's model can be derived from the probability distribution of the futures price without going through the complicated mathematical concepts like diffusion process or Brownian motion. For both the simulation and theoretical approach, in order to keep things simple, it was assumed that the option price was calculated for and at time **t** where **t** was equal to one. It's not a problem to calculate the option price for time period **t** at time 0 as long as **t** is equal to unity. The price calculated either by the simulation approach or the theoretical approach has to be discounted at the risk-free rate in order to calculate the option price for time period **t** at time 0. The difficulty arises when **t** is not equal to unity. The most important assumption used by Black is that, the variability in return of the futures is proportional to time **t**. Thus the variability in the price was adjusted for any time period through the return structure. If the price distribution is non-lognormal, it may either be difficult to figure out the return structure of the futures or it may be preferable to work with the price only. In such a situation, the variability in the price has to be adjusted for any time period t. It is not difficult to do so. A data set of the daily quotes of futures price for time period of **t** has to be found and the estimate of the variability in the price for time period **t** can be obtained from this data set. Once it has been done, then it is possible to compute the option prices for Beta, Gamma and Uniform distribution by either using the Simulation Approach or the Theoretical Approach by working with the prices only.

4. SAMPLE CALCULATIONS

The histogram of the generated prices for Lognormal, Beta, Uniform and Gamma distribution are shown in Figure 35, Figure 36, Figure 37 and Figure 38 respectively.

As it was discussed before, the current futures price "**pbarc**" and the volatility "**cvpc**" were known. Three levels of current futures price or **pbarc**, and three levels of futures volatility or **cvpc** were used. The levels of current futures prices were \$2.00, \$2.50 and \$3.00. The levels of volatility were 20%, 30% and 40%. Thus there were 9 possible combinations of current futures price and volatility of futures price.

Option prices were calculated for different distributions with strike prices ranging from \$0.00 to \$5.00 with an increment of \$0.25. The results have been presented in Tables 18-26.

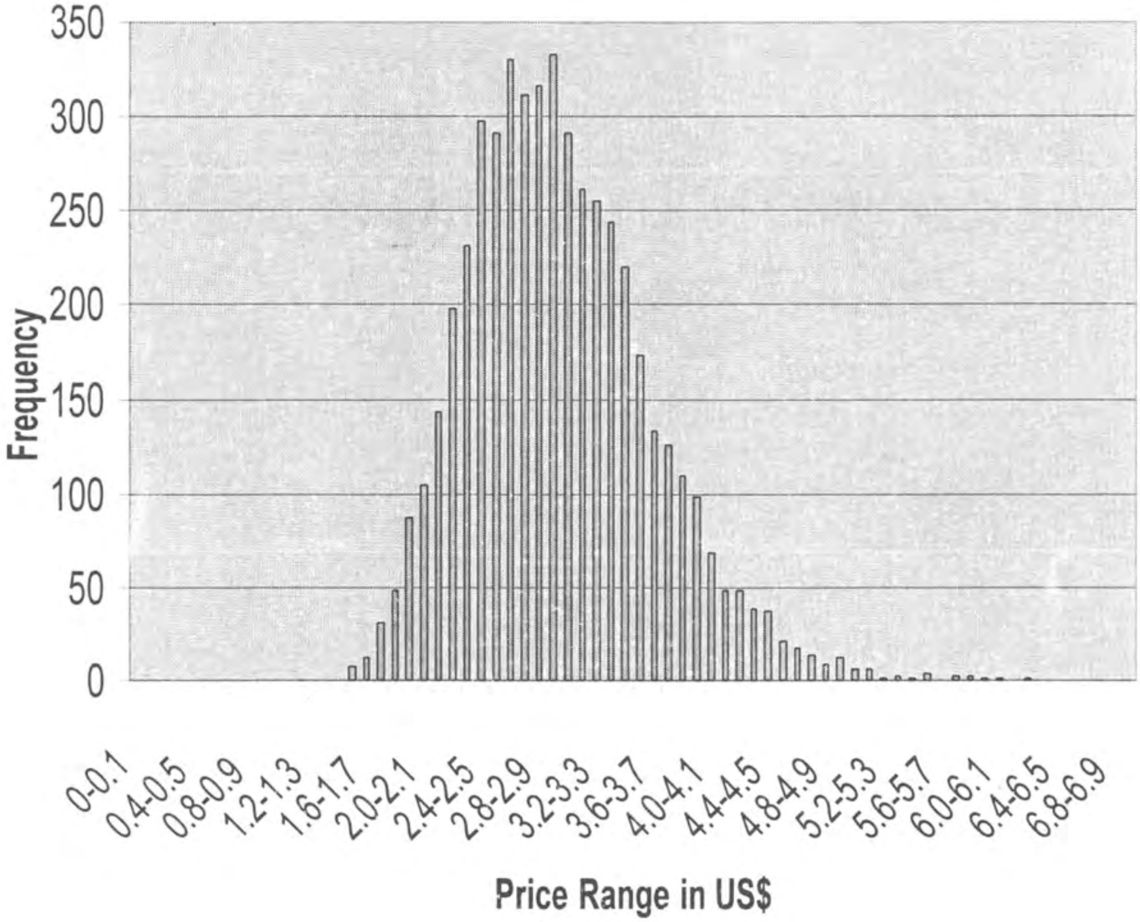

Figure 35. Histogram of Price for Lognormal Distribution

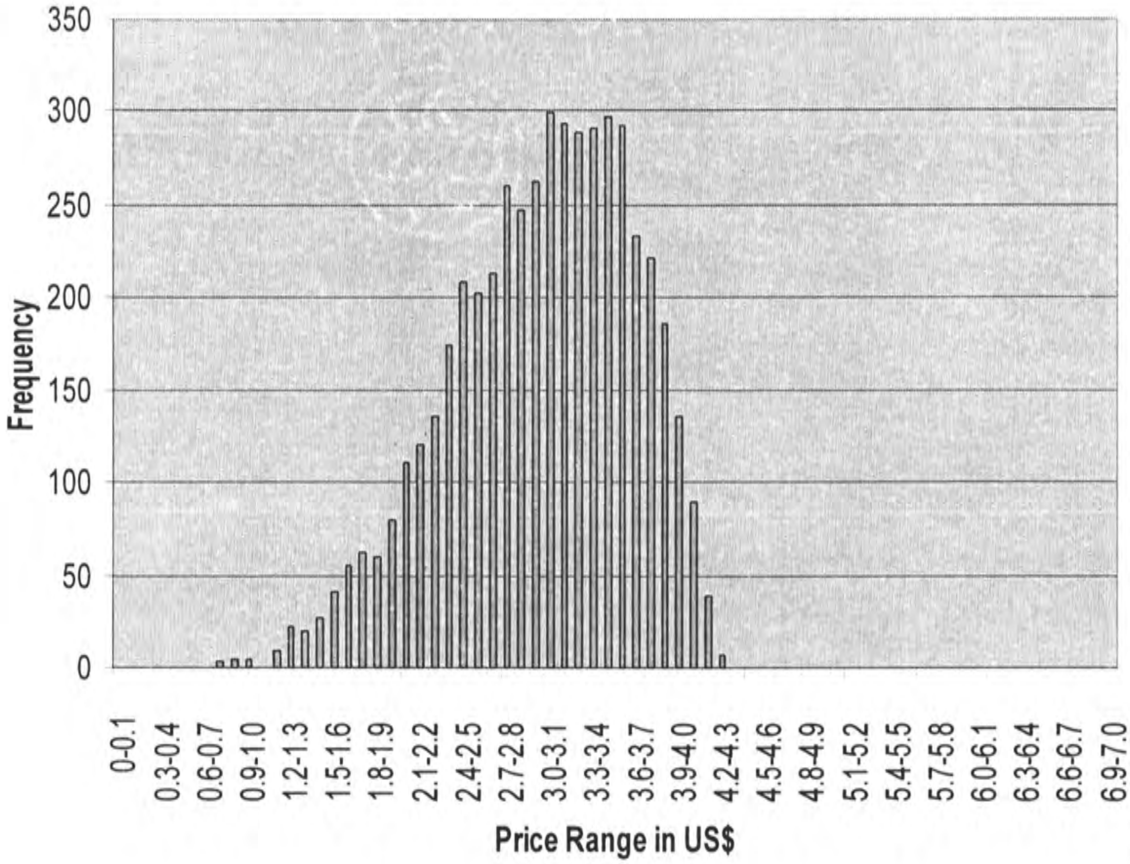

Figure 36. Histogram of Price for Beta Distribution

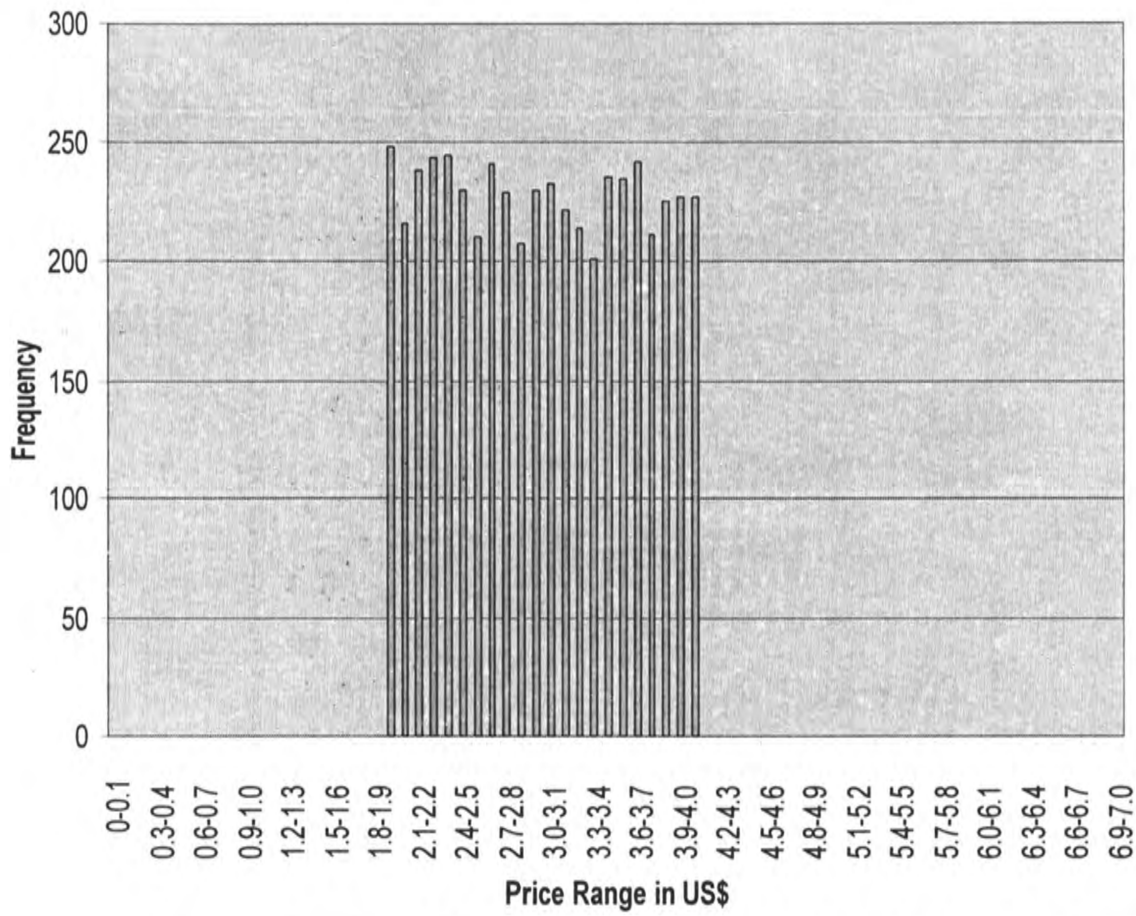

Figure 37. Histogram of Price for Uniform Distribution

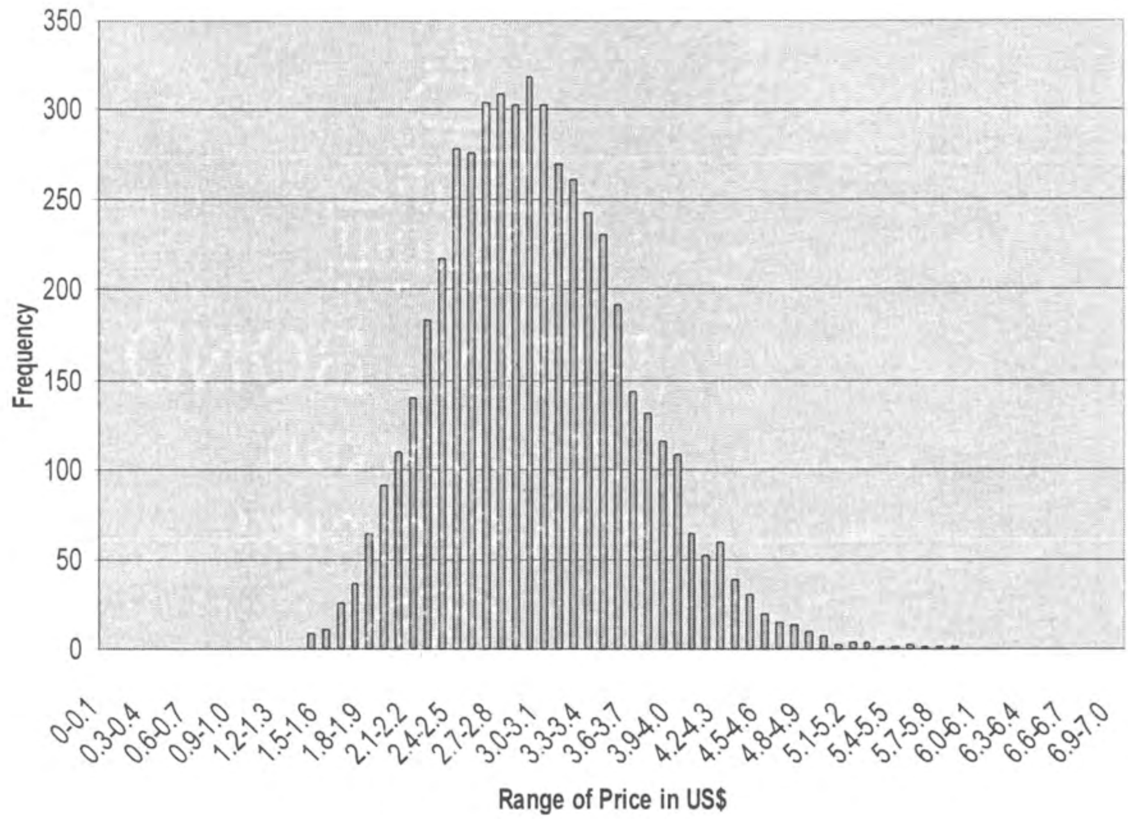

Figure 38. Histogram of Price for Gamma Distribution

Table 18. Option Prices for Futures price of $2.00 and Volatility of 20%

Strike	Lognormal	Beta	Uniform	Gamma
0.00	1.995095	1.991888	1.992879	1.994511
0.25	1.745095	1.741888	1.742879	1.744511
0.50	1.495095	1.491888	1.492879	1.494511
0.75	1.245095	1.242369	1.242879	1.244511
1.00	0.995108	0.995616	0.992879	0.994690
1.25	0.747205	0.756703	0.743083	0.748300
1.50	0.513497	0.533169	0.518453	0.517111
1.75	0.317423	0.337356	0.335635	0.321570
2.00	**0.176490**	**0.181250**	**0.192616**	**0.177980**
2.25	0.088726	0.073757	0.089016	0.087040
2.50	0.041495	0.017207	0.024538	0.038178
2.75	0.018215	0.000715	0.000209	0.015069
3.00	0.007589	0.000000	0.000000	0.005382
3.25	0.003226	0.000000	0.000000	0.001887
3.50	0.001374	0.000000	0.000000	0.000663
3.75	0.000572	0.000000	0.000000	0.000155
4.00	0.000156	0.000000	0.000000	0.000005
4.25	0.000000	0.000000	0.000000	0.000000
4.50	0.000000	0.000000	0.000000	0.000000
4.75	0.000000	0.000000	0.000000	0.000000
5.00	0.000000	0.000000	0.000000	0.000000

Table 19. Option Prices for Futures price of $2.00 and Volatility of 30%

Strike	Lognormal	Beta	Uniform	Gamma
0.00	1.9944646	1.9902200	1.9912787	1.9935770
0.25	1.7444646	1.7402200	1.7412787	1.7435770
0.50	1.4944646	1.4904518	1.4912787	1.4935770
0.75	1.2444646	1.2427139	1.2412787	1.2436039
1.00	0.9949642	1.0009823	0.9912787	0.9952032
1.25	0.7519032	0.7703529	0.7524075	0.7552382
1.50	0.5300308	0.5583687	0.5470113	0.5363083
1.75	0.3486327	0.3741163	0.3754308	0.3547460
2.00	**0.2148905**	**0.2239045**	**0.2359050**	**0.2176663**
2.25	0.1250212	0.1128321	0.1287890	0.1238404
2.50	0.0699224	0.0420156	0.0531030	0.0658516
2.75	0.0377753	0.0084925	0.0106499	0.0329114
3.00	0.0201156	0.0001844	0.0000000	0.0156766
3.25	0.0103696	0.0000000	0.0000000	0.0070072
3.50	0.0055009	0.0000000	0.0000000	0.0031679
3.75	0.0029183	0.0000000	0.0000000	0.0014102
4.00	0.0016112	0.0000000	0.0000000	0.0006045
4.25	0.0008529	0.0000000	0.0000000	0.0001782
4.50	0.0003906	0.0000000	0.0000000	0.0000000
4.75	0.0001111	0.0000000	0.0000000	0.0000000
5.00	0.0000000	0.0000000	0.0000000	0.0000000

Table 20. Option Prices for Futures price of $2.00 and Volatility of 40%

Strike	Lognormal	Beta	Uniform	Gamma
0.00	1.9940690	1.9888880	1.9899300	1.9928800
0.25	1.7440686	1.7388907	1.7399295	1.7428804
0.50	1.4940686	1.4898682	1.4899295	1.4928804
0.75	1.2440837	1.2448628	1.2399300	1.2432542
1.00	0.9959252	1.0085039	0.9922323	0.9975496
1.25	0.7585167	0.7855970	0.7686572	0.7643749
1.50	0.5470141	0.5829629	0.5748222	0.5559463
1.75	0.3755292	0.4065477	0.4098158	0.3837192
2.00	**0.2466885**	**0.2603395**	**0.2723997**	**0.2509561**
2.25	0.1560612	0.1475984	0.1632068	0.1557499
2.50	0.0961535	0.0685611	0.0810012	0.0920302
2.75	0.0584356	0.0228748	0.0276176	0.0525410
3.00	0.0347821	0.0033556	0.0021864	0.0286334
3.25	0.0207591	0.0000000	0.0000000	0.0152015
3.50	0.0121478	0.0000000	0.0000000	0.0078250
3.75	0.0073354	0.0000000	0.0000000	0.0040687
4.00	0.0044496	0.0000000	0.0000000	0.0020847
4.25	0.0027420	0.0000000	0.0000000	0.0010953
4.50	0.0017413	0.0000000	0.0000000	0.0004854
4.75	0.0010313	0.0000000	0.0000000	0.0001428
5.00	0.0005650	0.0000000	0.0000000	0.0000000

Table 21. Option Prices for Futures price of $2.50 and Volatility of 20%

Strike	Lognormal	Beta	Uniform	Gamma
0.00	2.4947553	2.4917977	2.4928791	2.4942965
0.25	2.2447553	2.2417977	2.2428791	2.2442965
0.50	1.9947553	1.9917977	1.9928791	1.9942965
0.75	1.7447553	1.7417977	1.7428791	1.7442965
1.00	1.4947553	1.4918625	1.4928791	1.4942965
1.25	1.2447553	1.2427991	1.2428791	1.2442965
1.50	0.9948665	0.9967653	0.9928791	0.9946541
1.75	0.7479309	0.7583723	0.7430832	0.7490764
2.00	0.5157573	0.5345908	0.5184533	0.5187273
2.25	0.3198591	0.3375049	0.3356347	0.3230138
2.50	**0.1772185**	**0.1798177**	**0.1926157**	**0.1781537**
2.75	0.0876181	0.0715474	0.0890159	0.0860108
3.00	0.0395433	0.0157964	0.0245384	0.0367254
3.25	0.0164255	0.0005642	0.0002085	0.0138988
3.50	0.0063394	0.000000	0.0000000	0.0046940
3.75	0.0024647	0.000000	0.0000000	0.0015260
4.00	0.0009678	0.000000	0.0000000	0.0004829
4.25	0.0003219	0.000000	0.0000000	7.022E-05
4.50	0.0000000	0.000000	0.0000000	0.0000000
4.75	0.0000000	0.000000	0.0000000	0.0000000
5.00	0.0000000	0.000000	0.0000000	0.0000000

Table 22. Option Prices for Futures price of $2.50 and Volatility of 30%

Strike	Lognormal	Beta	Uniform	Gamma
0.00	2.4939503	2.4900527	2.4912787	2.4932504
0.25	2.2439503	2.2400527	2.2412787	2.2432504
0.50	1.9939503	1.9900527	1.9912787	1.9932504
0.75	1.7439503	1.7400949	1.7412787	1.7432504
1.00	1.4939503	1.4908774	1.4912787	1.4932504
1.25	1.2439536	1.2441756	1.2412787	1.2434163
1.50	0.9951810	1.003453	0.9912787	0.9957875
1.75	0.7540712	0.7730368	0.7524075	0.7571438
2.00	0.5339063	0.5601469	0.5470113	0.5389466
2.25	0.3521526	0.3740427	0.3754308	0.3567158
2.50	**0.2162586**	**0.2218127**	**0.2359050**	**0.2180049**
2.75	0.1240878	0.1095699	0.1287890	0.1226462
3.00	0.0674652	0.0392174	0.0531030	0.0638183
3.25	0.0349839	0.0073039	0.0106499	0.0308795
3.50	0.0176298	0.0001306	0.0000000	0.0140750
3.75	0.0084942	0.0000000	0.0000000	0.0059858
4.00	0.0042005	0.0000000	0.0000000	0.0025317
4.25	0.0020634	0.0000000	0.0000000	0.0010705
4.50	0.0010517	0.0000000	0.0000000	0.0003873
4.75	0.0004523	0.0000000	0.0000000	7.243E-05
5.00	0.0001230	0.0000000	0.0000000	0.0000000

Table 23. Option Prices for Futures price of $2.50 and Volatility of 40%

Strike	Lognormal	Beta	Uniform	Gamma
0.00	2.4933814	2.4886280	2.4899295	2.4924388
0.25	2.2433814	2.2386280	2.2399295	2.2424388
0.50	1.9933814	1.9886280	1.9899295	1.9924388
0.75	1.7433814	1.7390371	1.7399295	1.7424388
1.00	1.4933814	1.4912516	1.4899295	1.4924523
1.25	1.2436212	1.2477756	1.2399295	1.2434250
1.50	0.9970670	1.0122896	0.9922323	0.9991909
1.75	0.7622880	0.7891723	0.7686572	0.7674959
2.00	0.5524606	0.5849569	0.5748222	0.5595130
2.25	0.3801769	0.4061683	0.4098158	0.3862285
2.50	**0.2487760**	**0.2576337**	**0.2723997**	**0.2515023**
2.75	0.1554840	0.1433841	0.1632068	0.1545255
3.00	0.0935577	0.0644086	0.0810012	0.0895862
3.25	0.0550045	0.0202259	0.0276176	0.0498072
3.50	0.0313368	0.0026001	0.0021864	0.0262058
3.75	0.0176957	0.0000000	0.0000000	0.0132503
4.00	0.0097575	0.0000000	0.0000000	0.0065210
4.25	0.0055334	0.0000000	0.0000000	0.0031953
4.50	0.0031109	0.0000000	0.0000000	0.0015584
4.75	0.0018124	0.0000000	0.0000000	0.0007379
5.00	0.0010228	0.0000000	0.0000000	0.0002769

Table 24. Option Prices for Futures price of $3.00 and Volatility of 20%

Strike	Lognormal	Beta	Uniform	Gamma
0.00	2.9945318	2.9917484	2.9928791	2.9941560
0.25	2.7445318	2.7417484	2.7428791	2.7441560
0.50	2.4945318	2.4917484	2.4928791	2.4941560
0.75	2.2445318	2.2417484	2.2428791	2.2441560
1.00	1.9945318	1.9917484	1.9928791	1.9941560
1.25	1.7445318	1.7417484	1.7428791	1.7441560
1.50	1.4945318	1.4919362	1.4928791	1.4941560
1.75	1.2445318	1.2431783	1.2428791	1.2441560
2.00	0.9947616	0.9975476	0.9928791	0.9946763
2.25	0.7485225	0.7594283	0.7430832	0.7496334
2.50	0.5172894	0.5354558	0.5184533	0.5197926
2.75	0.3214137	0.3375523	0.3356347	0.3239408
3.00	**0.1776028**	**0.1788403**	**0.1926157**	**0.1782376**
3.25	0.0867746	0.0700734	0.0890159	0.0852896
3.50	0.0381643	0.0148885	0.0245384	0.0357315
3.75	0.0152146	0.0004782	0.0002085	0.0131176
4.00	0.0055437	0.0000000	0.0000000	0.0042446
4.25	0.0020069	0.0000000	0.0000000	0.0013094
4.50	0.0007390	0.0000000	0.0000000	0.0003792
4.75	0.0001990	0.0000000	0.0000000	0.0000000
5.00	0.0000000	0.0000000	0.0000000	0.0000000

Table 25. Option Prices for Futures price of $3.00 and Volatility of 30%

Strike	Lognormal	Beta	Uniform	Gamma
0.00	2.9936106	2.9899624	2.9912787	2.9930363
0.25	2.7436106	2.7399624	2.7412787	2.7430363
0.50	2.4936106	2.4899624	2.4912787	2.4930363
0.75	2.2436106	2.2399624	2.2412787	2.2430363
1.00	1.9936106	1.9899624	1.9912787	1.9930363
1.25	1.7436106	1.7401777	1.7412787	1.7430363
1.50	1.4936106	1.4913956	1.4912787	1.4930363
1.75	1.2436800	1.2452626	1.2412787	1.2433501
2.00	0.9955030	1.0050423	0.9912787	0.9962838
2.25	0.7556989	0.7746793	0.7524075	0.7584314
2.50	0.5364812	0.5611901	0.5470113	0.5406595
2.75	0.3543611	0.3739066	0.3754308	0.3579661
3.00	**0.2169824**	**0.2203774**	**0.2359050**	**0.2181780**
3.25	0.1232723	0.1073843	0.1287890	0.1217868
3.50	0.0656538	0.0373758	0.0531030	0.0624071
3.75	0.0330143	0.0065631	0.0106499	0.0294957
4.00	0.0159458	0.0001032	0.0000000	0.0130099
4.25	0.0072958	0.000000	0.0000000	0.0053187
4.50	0.0034088	0.000000	0.0000000	0.0021408
4.75	0.0015787	0.000000	0.0000000	0.0008722
5.00	0.0007224	0.000000	0.0000000	0.0002781

Table 26. Option Prices for Futures price of $3.00 and Volatility of 40%

Strike	Lognormal	Beta	Uniform	Gamma
0.00	2.9929252	2.9884885	2.9899295	2.9921499
0.25	2.7429252	2.7384885	2.7399295	2.7421499
0.50	2.4929252	2.4884885	2.4899295	2.4921499
0.75	2.2429252	2.2384885	2.2399295	2.2421499
1.00	1.9929252	1.9885952	1.9899295	1.9921499
1.25	1.7429252	1.7394582	1.7399295	1.7421499
1.50	1.4929252	1.4924097	1.4899295	1.4922441
1.75	1.2435124	1.2497461	1.2399295	1.2437075
2.00	0.9981926	1.0146455	0.9922323	1.0004046
2.25	0.7650289	0.7913117	0.7686572	0.7695727
2.50	0.5560298	0.5860689	0.5748222	0.5618055
2.75	0.3830702	0.4057830	0.4098158	0.3878087
3.00	**0.2499191**	**0.2557546**	**0.2723997**	**0.2517801**
3.25	0.1548156	0.1405443	0.1632068	0.1536139
3.50	0.0915449	0.0616586	0.0810012	0.0878661
3.75	0.0524943	0.0185263	0.0276176	0.0479172
4.00	0.0289181	0.0021652	0.0021864	0.0245638
4.25	0.0156153	0.0000000	0.0000000	0.0119700
4.50	0.0082525	0.0000000	0.0000000	0.0056968
4.75	0.0044387	0.0000000	0.0000000	0.0026609
5.00	0.0023631	0.0000000	0.0000000	0.0012553

5. CONCLUSIONS

It can be seen from the attached tables that lognormal distribution underprices in-the-money and at-the-money options but overprices out-of-the money options compared to the other three distributions. The lognormal distribution also overprices the deep-in-the-money or deep-out-of-the-money options. If the true distribution is one of those three but the market uses lognormal distribution then the best strategy would be to sell out-of-the money options and buy in-the money options or at-the money options.

The contribution of this paper is threefold. First, it shows how, using the live hog futures data, the distribution of futures price varies from the assumed lognormal distribution. Second, it shows how the convenient, simple and open-architecture EXCEL simulation technique can be used to accommodate any probability distribution of the underlying asset and calculate the value of the option. Third, it shows how, with a little adjustment, the value of the option can be computed theoretically when the underlying asset has Beta, Gamma or Uniform probability distribution.

REFERENCES

Bachelier, L (1900) Theorie de la speculation In P. Cootner (ed.), *The Random Character of Stock Market Prices*. Cambridge, Mass., MIT Press.

Black, Fischer and Scholes, Myron (1973), "The Pricing of Options and Corporate Liabilities", *Journal of Political Economy* 81, 637-659.

Black, Fischer (1976), "The Pricing of Commodity Contracts", *Journal of Financial Economics* 3, 167-179.

Casella, George and Berger, Roger. L. (1990), *Statistical Inferences, 1st ed.,* California, Duxbury Press.

Cootner, Paul (1960), "Returns to Speculators: Telser versus Keynes", *Journal of Political Economy* 67, 396-418.

Dusak, K. (1973), "Futures Trading and Investor Returns: An Investigation of Commodity Market Risk Premiums", *Journal of Political Economy* 81, 1387-1406.

Eberlein, Ernst and Keller, Ulrich (1995), "Hyperbolic distributions in finance", *Bernoulli* 1, 281-299.

Eberlein, Ernst and Keller, Ulrich and Prause, Karsten (1998), "New Insights into Smile, Mispricing, and Value at Risk: The Hyperbolic Model", *Journal of Business* 71, 371-405.

Fama, Eugene (1965), "The behavior of stock market prices", *Journal of Business* 38, 34-105.

Hardy, Charles O. (1940), *Readings in Risk and Risk Bearing*, Chicago, Univ. Chicago Press.

Hull, J.C. (1998), *Introduction to Futures and Options, 3rd ed.,* New Jersey, Prentice Hall.

Hull, J. and White, A. (1987), "The Pricing of Options on Assets with Stochastic Volatilities", *The Journal of Finance* XLII,

Jarrow, R. and Turnbull, S. (2000), *Derivative Securities*, *2nd ed.,* Cincinnati, South-Western.

Johnson, N.L. and Kotz, S. and Balakrishnan N. (1994), *Continuous Univariate Distributions*, *2nd ed., Volume 1,* New York City, John Wiley & Sons, Inc.

Johnson, N.L. and Kotz, S. and Balakrishnan N. (1995), *Continuous Univariate Distributions*, *2nd ed., Volume 2,* New York City, John Wiley & Sons, Inc.

Keynes, J.M. (1930), *A Treatise on Money*, *Volume 2,* London, Macmillan.

Mandelbrot, Benoit (1963), "The variation of certain speculative prices", *Journal of Business* 36, 394-419.

Osborne, M. F. M. (1959) "Brownian Motion in the stock market", *Operations Research* 7, 145-173.

Samuelson, P. (1965) "Rational theory of warrant pricing", *Industrial Management Review* 6, 13-32.

Smith, Clifford, W., Jr. (1976), "Option Pricing, A review* ", *The Journal of Financial Economics* 3, 3-51.

Stevenson, Richard A. and Bear, Robert M. (1970), "Commodity Futures: Trends or Random Walks?", *Journal of Finance* 25, 65-81.